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Single sideband, suppressed carrier radio communication.

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SINGLE SIDEBAND, SUPPRESSED CARRIER RADIO COMMUNICATION

BY

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SINGLE-SIDEBAND, SUPPRESSED CARRIER RADIO COMMUNICATION

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SINGLE SIDEBAND, SUPPRESSED CARRIER
RADIO COMMUNICATION

by

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PREFACE

In 1923, the first radio communications link employing modulation in which the carrier and one sideband had been eliminated was established. Since then, the use of this system of transmission has grown to a considerable extent. The basic advantages of this system of communications are great. The major problems of the system have been overcome, although some problems do remain. And, too, there are some definite limitations to the practical applications of the system.

Because the writer believes this system holds much promise in the near future, this study of the nature and practical aspects of single sideband, suppressed carrier communications has been prepared.

The faculty of the Electrical Engineering Department of the United States Naval Postgraduate School has been extremely helpful in the preparation of this paper, and to them, the writer wishes to express his most sincere appreciation. In particular, he desires to thank Professor C. V. O. Terwilliger, chairman of the department, and Professor W. C. Smith, without whose guidance, assistance and encouragement this paper would have been impossible to prepare.

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LIST OF ILLUSTRATIONS

TABLE OF SYMBOLS AND ABBREVIATIONS

These are listed in the order of their use in the text.

A.M. -- Amplitude Modulated

F.M. -- Frequency Modulated

P.M. -- Phase Modulated

KC -- Kilocycles

e -- Instantaneous Voltage

M -- Modulation Factor (Degree of Modulation)

ω -- 2π Times Frequency in Cycles per Second

ω_s -- 2π Times Signal Frequency

ω_c -- 2π Times Carrier Frequency

e_c -- Instantaneous Carrier Voltage

e_s -- Instantaneous Signal Voltage

e_o -- Instantaneous Resultant Voltage

SSSC -- Single-Sideband, Suppressed Carrier

SSWC -- Single-Sideband, With Carrier

SSRC -- Single-Sideband, Reduced Carrier

R.F. -- Radio Frequency

i_p -- Instantaneous Plate Current

R -- Resistance

i -- Instantaneous Current

e_g -- Instantaneous Grid Voltage

e_p -- Instantaneous Plate Voltage

CW -- Carrier Wave (Code)

BFO -- Beat-Frequency Oscillator

C.P.S. -- Cycles per Second

KW -- Kilowatts

CHAPTER I

WHAT IS SINGLE SIDEBAND SUPPRESSED CARRIER COMMUNICATIONS?

This paper concerns itself with a type of modulation of a radio-frequency carrier wave. Modulation, as is very well known, is the process of introducing intelligence into a fixed frequency, constant amplitude carrier wave. Several methods for accomplishing this end are in general use, each having it's advantages and limitations. A brief discussion of the most common modulation methods is in order at this time.

Three methods of modulation are widely used in radio communications. These are amplitude modulation (A.M.), frequency modulation (F.M.), and phase modulation (P.M.). The first, A. M., is the most commonly used method. All radio broadcasting stations in the 550 KC to 1700 KC band use A.M. Most communications traffic also uses this modulation method. In principle and practice, this method of modulation is most simple, thus accounting for the widespread use. A.M. is usually obtained by superimposing the desired audio signal on the plate power supply of a non-linear, radio-frequency amplifier. The resultant output from this stage is a signal of variable amplitude, which may be represented by the equation,

$$e = A(1 + M \cos \omega_s t) \cos \omega_c t \qquad \text{(Eq. I)}$$

in which M is the modulation factor (degree of modulation, from 0 to 1), ω_s is the signal, or audio frequency, and ω_c is the carier frequency. Equation I is derived as follows:

$e_c = A \cos \omega_c t =$ carrier

$e_s = B \cos \omega_s t =$ modulating signal

$e_o = (1 + e_s)e_c = A(1 + B \cos \omega_s t) \cos \omega_c t.$

It will be noticed that the degree, or percent, of modulation is dependent on the value of B. Maximum, or 100% modulation will occur when the minimum amplitude of the modulated wave is zero. (A greater value of B will result in "over-modulation" which produces distortion and spurous ratiation. It is not acceptable.) This value will occur when B - 1, so that B may have a value from 0 to 1. M, the percent modulation, is defined as equal to B, thus

$$e_o = A(1 + M \cos \omega_s t) \cos \omega_c t \qquad \text{(Eq. I)}$$

Expanding this equation,

$$e_o = A \cos \omega_c t + 1/2\, AM \cos (\omega_c + \omega_s)t$$
$$+ 1/2\, AM \cos (\omega_c - \omega_s)t. \qquad \text{(Eq. II)}$$

Thus it is seen that an A. M. transmission contains components of three frequencies, the carrier, and the sum and difference of the carrier and signal frequencies. These sum and difference frequencies are known respectively, as upper and lower sidebands. Figure 1 may assist in visualizing A. M.

Frequency modulation is the next most popular method of modulation. In the last decade, it has become very popular because of certain inherent favorable characteristics. A widely used method of obtaining F. M. is the use of a "reactance tube" shunting the tank circuit of an oscillator. The output may be represented by

$$e_o = A \sin \left(\omega_c t + \frac{2\pi \Delta f}{\omega_s} \sin \omega_s t\right)$$

Where Δf is the maximum deviation of the instantaneous frequency from the carrier frequency and is determined by the peak magnitude of the modulating signal. It is independent of the modulating frequency. The expansion of this equation, while at first glance may appear to contain components of only a limited number of frequencies, actually contains an infinite number of frequencies, as is seen in the expanded equation below.

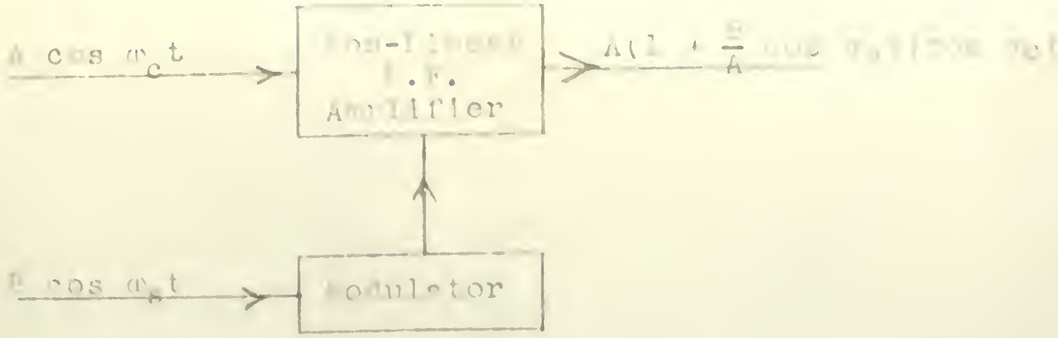


FIGURE 1(A). Block Diagram of
A.M. Modulator

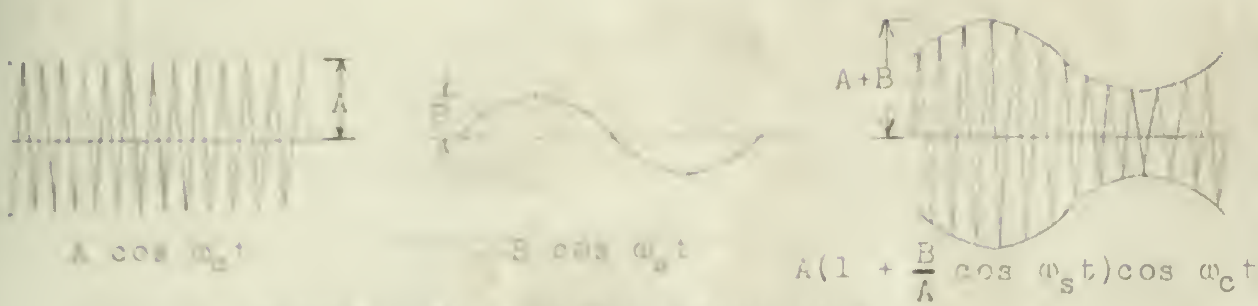


FIGURE 1(B). Single Modulating
Frequency Representation of A. M.

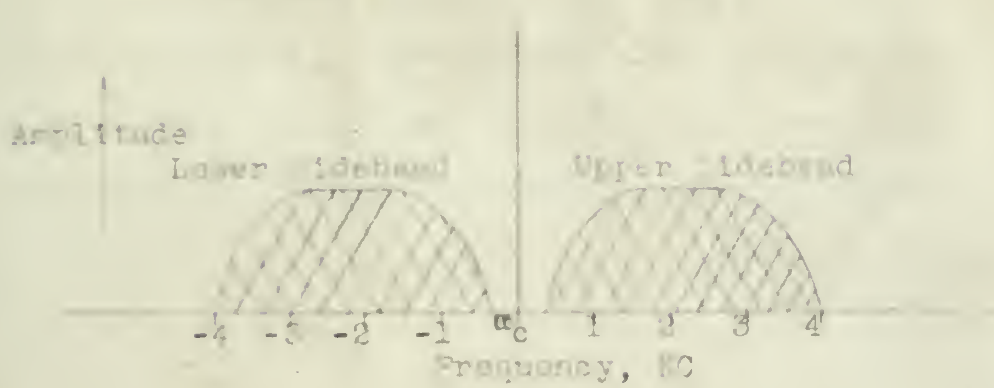


FIGURE 1(C). Representation of
Spectrum of A. M. with Finite Audio Range

$$
\begin{aligned}
e_o = A \Big\{ & J_o(m_f)\sin\omega_c t \\
& + J_1(m_f)\left[\sin(\omega_c+\omega_s)t - \sin(\omega_c-\omega_s)t\right] \\
& + J_2(m_f)\left[\sin(\omega_c+2\omega_s)t + \sin(\omega_c-2\omega_s)t\right] \\
& + J_3(m_f)\left[\sin(\omega_c+3\omega_s)t - \sin(\omega_c-3\omega_s)t\right] \\
& + \cdots\cdots\cdots\cdots \Big\}
\end{aligned}
$$

where $m_f = 2\pi\Delta f/\omega_s$ = modulating index, and $J_n(m_f)$ means the Bessel function of the first kind and nth order, with argument m_f. Thus it is seen that the frequency spread is wide, indeed. In actual practice, however, the frequency spread is approximately ($\omega_s + \Delta f$) on each side of the carrier frequency, with components spaced at frequency intervals that are equal to the modulating frequency.

Phase modulation is similar to frequency modulation in many respects. The equations of e_o are the same for phase modulation as for frequency modulation. The only difference is in the meaning of m, which for phase modulation should have a subscript of p. M_p is the angle in radians through which the phase is displaced, at the peak of the modulation cycle, away from the phase that would exist if there were no modulation.

This paper is not concerned with frequency or phase modulation, and no further mention will be made of them. Returning to Eq. II, it is again noted that e_o consists of three components. The carrier, of course, contains no intelligence. Each of the sidebands, however, contains all of the intelligence. It would seem logical, then, that if only the sidebands, or even only one sideband, were transmitted, all the intelligence would be contained. The immediate value of this type of transmission would be to greatly reduce the width of the transmission band for a given range of intelligence frequencies. This has many advantages over the other types of transmissions which, for certain types of communications, make it well worth the practical difficulties encountered in the production and reception of this type of transmission. This is Single-Sideband, Suppressed Carrier Communications (SSSC). Visualization of the waveform of SSSC is somewhat difficult, but Figure 2, which illustrates this with an excessively low carrier, may assist. Under certain conditions, it is desirable to transmit all or part of the carrier. This mode of operation will be called Single Sideband With Carrier (SSWC), or Single Sideband, Reduced Carrier (SSRC). A third possibility is the transmission of A.M. with only the carrier suppressed. Transmission in this manner is possible, and is in use for some applications, but involves additional reception problems which will be discussed later.

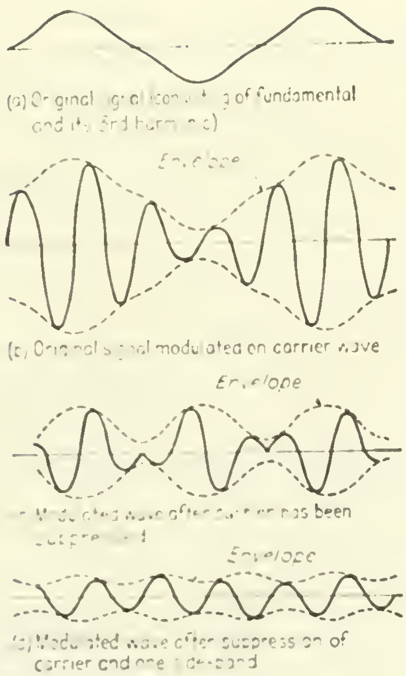


FIGURE 2. Character of waves produced when carrier is suppressed and when carrier and one side band are both suppressed. For purposes of clarity, the carrier frequency is made excessively low. 2

This paper will be a discussion of the nature of, and practical problems encountered in the transmission and reception of SSSC, SSWC, and SSRC. Before going into the details of SSSC operation, the necessary enthusiasm may be aroused by a quick look at some of the advantages. If a pure resistive load is assumed for the transmitter, the power output of the transmitter is E_o^2R. For an A.M. signal,

$$e_o = A \cos \omega_c t + AM/2 \cos (\omega_c + \omega_s)t$$
$$+ AM/2 \cos (\omega_c - \omega_s)t$$
$$E_o^2 = A^2 + (AM/2)^2 + (AM/2)^2 .$$

Thus the power output,

$$P_o = \left[1 + (M/2)^2 + (M/2)^2\right] RA^2 .$$

The power distribution among the three components of the A.M. output is:

Carrier $= (1 \cancel{- M^2/2})100\% / (1 + \frac{M^2}{2})$

Upper Sideband $= (M^2/4)100\% / (1 + \frac{M^2}{2})$

Lower Sideband $= (M^2/4)100\% / (1 + \frac{M^2}{2})$

For two values of M, the power distribution is:

	M = 1	M = .25
Carrier	66.7%	96.8%
Each Sideband	16.7%	1.56%

As mentioned earlier, overmodulation produces spurous frequency emittions. This is not only undesirable, but

is also illegal. To reduce the possibility of overmodulation on peaks of audio power, the percent modulation is normally kept at a much lower value than 100%. A value of 25% is normal. But at 25% modulation, the percent useful power is only 1.56%! This is only 780 watts useful power out of 50,000 watts transmitted power.! As one author[13] aptly put it, of this 50,000 watts of steam being generated, 48,400 watts is being used to toot the whistle!

The greatest improvement in this situation is realized by elimination of the carrier. Immediately, the percent power in each sideband becomes 50%. This is a feasible method of operation and is used to a great degree in wired communication systems. As will be explined in the chapter under Reception, this method of transmission introduces considerable difficulty in reception. Elimination of one sideband in addition to the carrier not only raises the percent usable power to 100%, but greatly reduces the reception problems introduced by carrier suppression. An advantage of even greater importance in many applications is the reduction of required frequency by half. The available frequency spectrum is not unlimited. With the constantly increasing demands for transmission space, some method of doubling the available space is not be be viewed lightly. For wired communication circuits, the spectrum is also limited by the physical constants of the transmission line.

The narrower the required band, the larger will be the number of messages per circuit. This method of modulation is most widely used by telephone companies, as would be expected.

Another not-too-obvious advantage of this system is the savings in transmitter cost. Most high-powered transmitters are modulated by plate modulating the final power amplifier. To accomplish this, an audio system is required that delivers half the input power to the final power amplifier. For a 50,000 watt transmitter, this means an audio power of approximately 25,000 watts is required. Audio equipment in this power classification will be a major item in the overall cost of the transmitter. When SSSC is employed, modulation takes place in the lowest powered stages. Receiving-type equipment may be used for all the audio equipment, thus greatly reducing the equipment cost.

There are other advantages in the use of SSSC, and they will be discussed later.

CHAPTER II

SSSC TRANSMISSION

The problem of generation of SSSC resolves itself into separate stages. First is the elimination of the carrier. Next is the suppression of one of the sidebands. In some circuits, the order of these stages is reversed or even split, but the two stages are always distinct. This process may be represented by the block diagram of Figure 3.

The oscillator, or oscillators (as it will be seen later, several oscillators may be required) will not be discussed at this time except to mention that they must be of very stable design.

The modulator and carrier suppressor is so listed, because the use of a single unit of proper type will accomplish both requirements. Several types of units are used for this purpose. The first, and most commonly used, is the balanced modulator. Figure 4 is a simplified schematic diagram of a balanced modulator. Consideration of this circuit, however, will be made easier by first considering Figure 5, the Van der Bijl modulator.

In the Van der Bijl modulator, as in the balanced modulator, the circuit is that of a Class A, R. F. amplifier with the audio signal input fed into the stage in series with the carrier frequency input. It will be

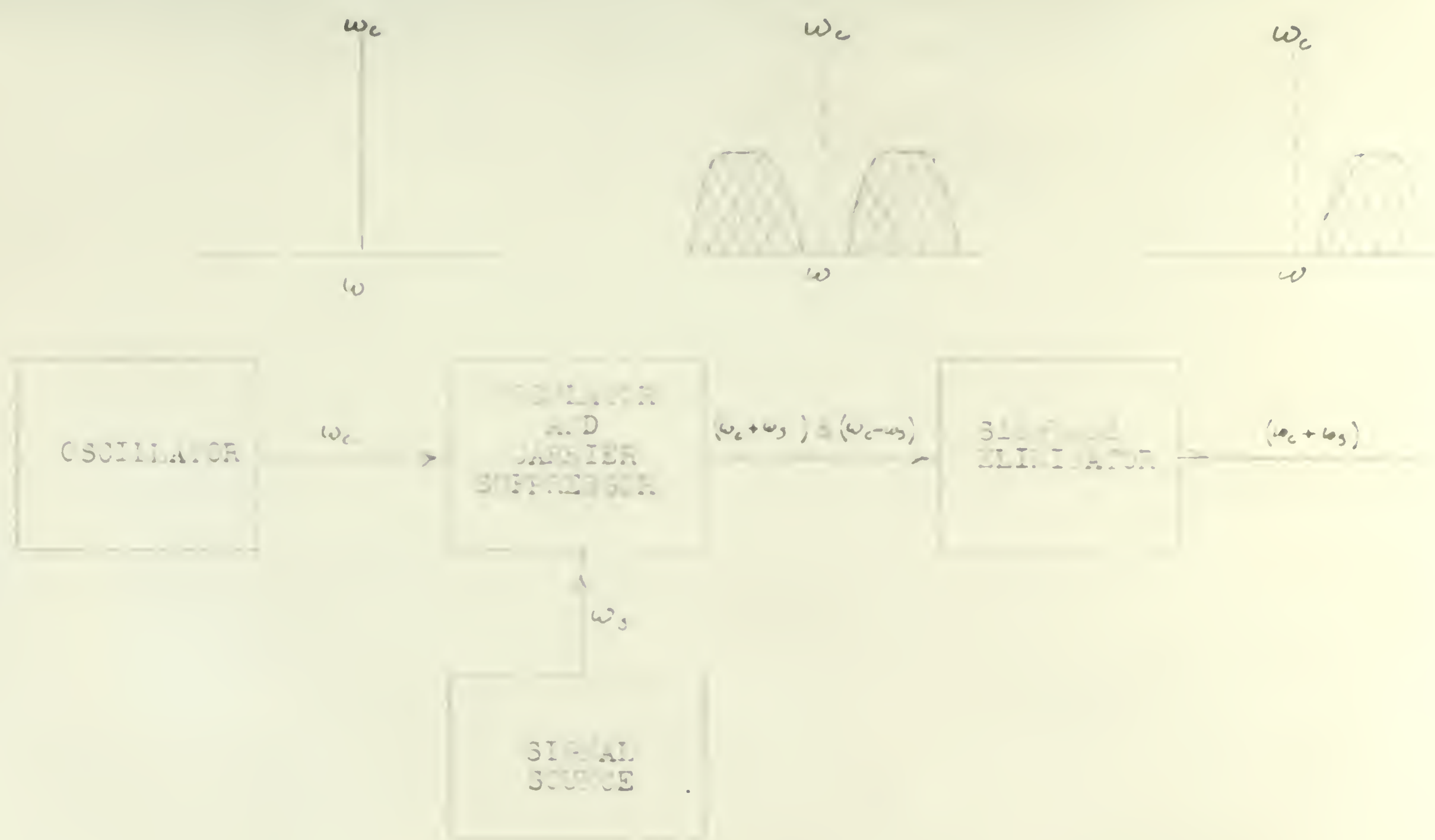


FIGURE 3. PROCESS OF PRODUCTION OF SSSC WAVE.

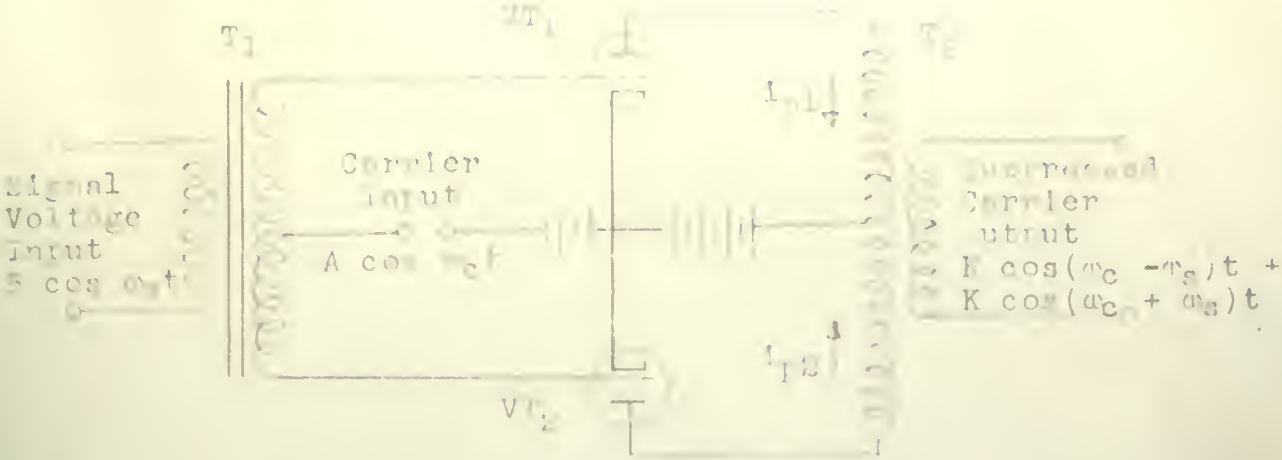


FIGURE 4. The Balanced Modulator

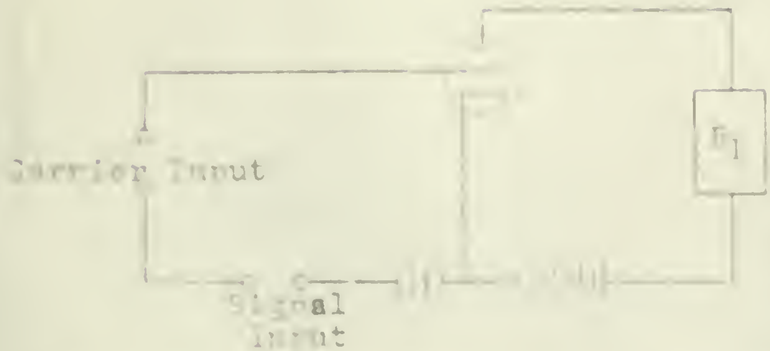


FIGURE 5. The Van der Bijl Modulator

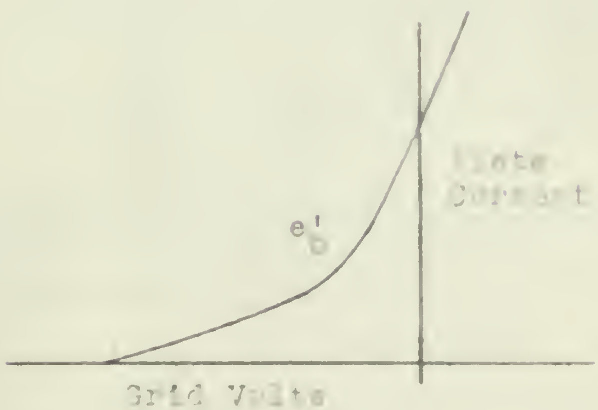


FIGURE 6. Typical Triode Characteristics

remembered that the characteristics of a triode are non-linear, as indicated by Figure 6. This curve may be expressed as a power series. Considering only the first two terms of this power series,

$$i_p = a_1 e + a_2 e^2 = \text{plate, or load current}$$

where

$$e = e_c + e_s = B \cos \omega_s t + A \cos \omega_c t = \text{exciting voltage.}$$

Then:

$$i_p = a_1 (A \cos \omega_c t + B \cos \omega_s t)$$
$$+ a_2 (A \cos \omega_c t + B \cos \omega_s t)^2.$$

By expansion and trigonometric rearrangement,

$$i_p = a_1 A (1 + \frac{2a_2}{a_1} B \cos \omega_s t) \cos \omega_c t + a_1 B \cos \omega_s t$$
$$+ a_2 B^2 \cos^2 \omega_s t + a_2 A^2 \cos^2 \omega_c t.$$

Now note that the first term of the last equation is identical in form to Eq. I, which is the equation of an amplitude modulated wave. The remaining three components are removed from the circuit by suitable filtering.

Returning to the balanced modulator of Figure 4, it is seen to be a straightforward push-pull stage of Class A amplification to which a source of carrier voltage has been added. Considering the audio source only, it is seen that it is amplified in a conventional manner and will appear across the primary of T_2. The carrier voltage, however, is impressed on the grids of VT_1 and VT_2 in the same phase, producing currents, as indicated in Figure 4, in the same direction with respect to the plates of the tubes. These currents induce voltages in T_2 that cancel, thus the carrier does not appear in the output. The sidebands, $kAB \cos(\omega_c + \omega_s)t$ and $kAB \cos(\omega_c - \omega_s)t$ will appear, however. This may be shown mathematically as follows:

Exciting voltage of $VT_1 = e_c + e_s$

Exciting voltage of $VT_2 = e_c - e_s$

$$i_{p1} = a_1(e_c + e_s) + a_2(e_c + e_s)^2 + a_3(e_c + e_s)^3 \ldots\ldots\ldots$$

$$i_{p2} = a_1(e_c - e_s) + a_2(e_c - e_s)^2 + a_3(e_c - e_s)^3 \ldots\ldots\ldots$$

Note, now, that i_{p1} and i_{p2} flow in the primary of T_2 in opposite directions, so that the voltage induced in the secondary is proportional to $i_{p1} - i_{p2}$.

$$e_o = K(i_{p1} - i_{p2})$$
$$= 2A\Big[a_1e_s + 2a_2e_ce_s + a_3(3e_c^2e_s + e_s^3) + a_4(4e_c^3e_s + 4e_ce_s^3) + a_5(5e_c^4e_s + 10e_c^2e_s^3 + e_s^5) + \ldots\ldots\ldots\Big].$$

Now, a_3, a_4, and a_5 are small in comparison with a_1 and a_2. Also, the third and higher order terms become smaller as e_s, or percent modulation, becomes smaller. The effect of all terms above the second order may thus be made negligible. The equation then reduces to

$$e_o = 2K(a_1e_s + 2a_2e_ce_s).$$

As $e_c = A \cos \omega_c t,$, and

$$e_s = B \cos \omega_s t,$$
$$e_o = 2Ka_1B(1 + 2\frac{a_2}{a_1} A \cos \omega_c t)\cos \omega_s t$$

which, by trignometric rearrangements, gives

$$e_o = 2Ka_1B \cos \omega_s t + 1/2\Big[4Ka_2AB \cos(\omega_c + \omega_s)t + 4Ka_2AB \cos(\omega_c - \omega_s)t\Big].$$

Note that this output contains only the sidebands plus the modulating signal frequency component. This latter component is attentuated sufficiently in T_2 so as to be negligible.

In this derivation, perfect balance of tube and circuit elements has been assumed. In practice, it is usually necessary to provide adjustable balancing elements to the circuit, as well as to use care in selection of balanced tubes.

The functions of carrier suppression and modulation may also be accomplished by means of a circuit known as the "ring" modulator. As shown in Figure 7(A), this consists of four diode rectifiers, usually of the contact type, connected in a series ring. The nature of most diodes is such that with application of a forward voltage, the effective resistance R_f, will be small. With the application of a reverse voltage, the effective resistance, R_r, will be large. The magnitude of e_c is made very much greater than e_s, so that in effect, e_c is simply a switch that switches resistances between the positions in the equivalent circuits of Figure 7(B) and (C). It may readily be shown that efficiency is greatest for $R_S = R_s = R_L = \sqrt{R_f R_r} = R$. To simplify the following work, it will be assumed that this condition exists. Noting that $i_a \doteq e_s/2R$, the solution of the network for i_b becomes:

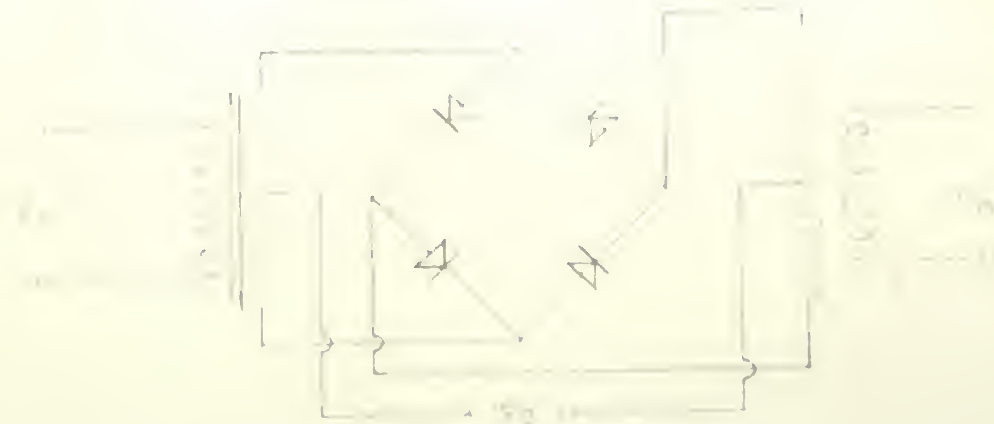


FIGURE 7(A). The Ring Modulator

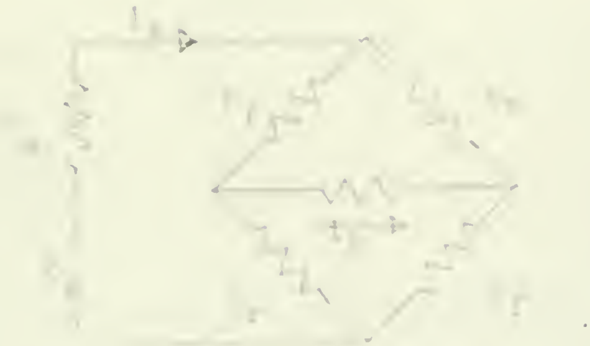


FIGURE 7(B). Equivalent Circuit of Ring Modulator For Conduction in One Direction

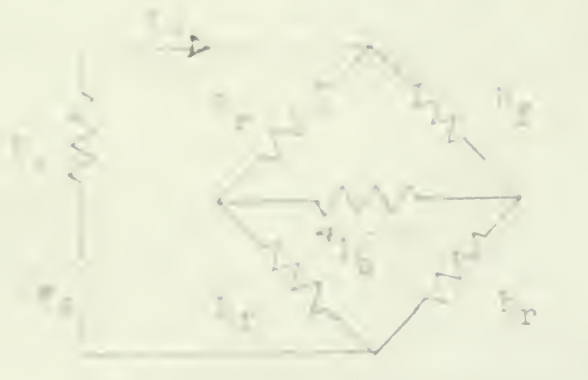


FIGURE 7(C). Equivalent Circuit of Ring Modulator For Conduction in Other Direction

$$i_b = \frac{e_s}{2R} \frac{\sqrt{\frac{R_r}{R_f}} - 1}{\sqrt{\frac{R_r}{R_f}} + 1} f(t) \quad ,$$

where $f(t) = \begin{cases} +1 \text{ for positive half cycle of carrier} \\ -1 \text{ for negative half cycle of carrier.} \end{cases}$

Representing $f(t)$ by Fourier components,

$$f(t) = \frac{4}{\pi} (\cos \omega_c t - \frac{1}{3} \cos 3 \omega_c t + \frac{1}{5} \cos 5 \omega_c t - \ldots\ldots).$$

Substituting $e_s = B \cos \omega_s t$.

$$i_b = \frac{4B}{2R} \frac{\sqrt{\frac{R_r}{R_f}} - 1}{\sqrt{\frac{R_r}{R_f}} + 1} \cos \omega_s t (\cos \omega_c t - \frac{1}{3} \cos 3\omega_c t \quad \ldots.).$$

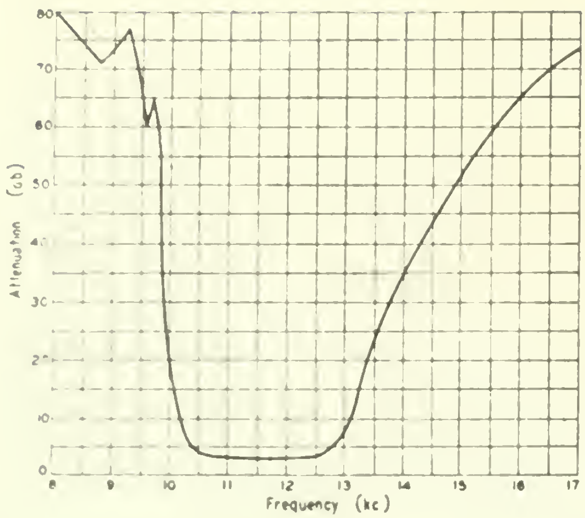
Now, as B is very small, the use of simple circuit elements will reduce components over the first order to a negligible value, leaving

$$i_b \doteq K \cos \omega_s t \cos \omega_c t$$

$$= \frac{K}{2} \cos(\omega_c + \omega_s)t + \frac{K}{2} \cos(\omega_c - \omega_s)t.$$

Thus, not only has the carrier been eliminated, but also so has the modulating signal.

The next stage of the transmitter is the sideband eliminator. Only two methods are in general use for this job. The most popular is the straight filter method. The sideband eliminator consists of a suitable filter to stop the unwanted sideband. This is known as the "brute force" type of sideband eliminator to contrast it with the more subtile, but also more critical and delicate, phasing method of sideband elimination. Figure 8 shows the necessary frequency characteristics of a suitable filter. The required sharpness is not a percent function of the carrier frequency, but is a function of the number of cycles per second. This would imply that the lower the carrier frequency, the less would be the required percent sharpness, and thus the simpler would be the filter design. In practice, if the initial carrier frequency is kept below about 40 KC, a conventional, multiple, M-derived band-pass filter is adequate. Figure 8 is the characteristic curve of such a filter using a 10KC initial carrier frequency. If the initial carrier frequency exceeds 40 KC, more complex filters of the crystal-lattice type are required. A ceiling of approximately 100 to 150 KC exists for practical filters. Discussed in this paper will be only filters for use with carrier frequencies up to 40KC. This low frequency carrier modulated by a balanced modulator, which eliminates the carrier, then one sideband



(National Company, Inc.)

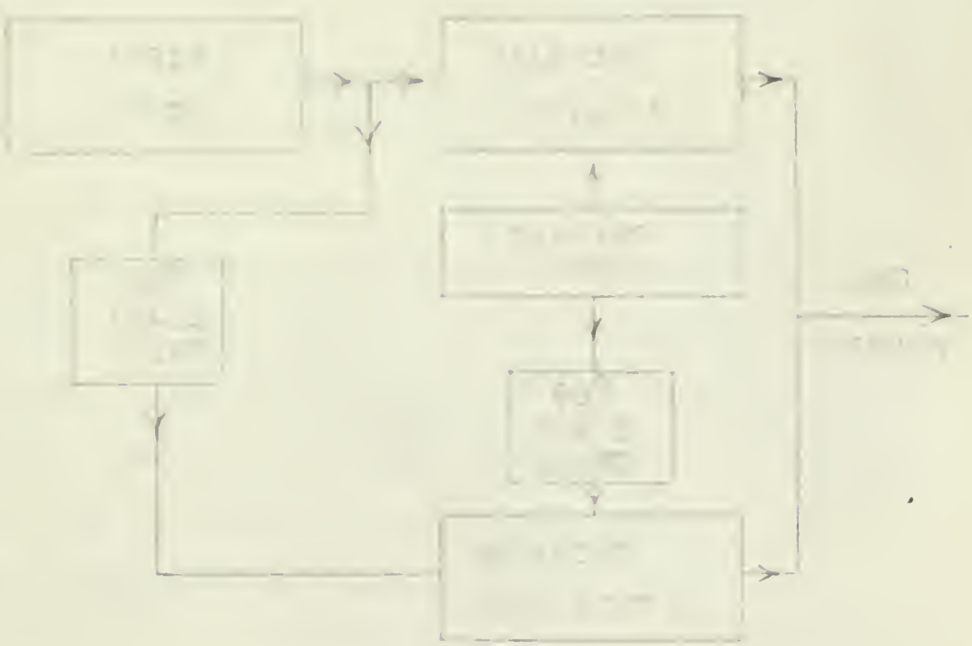
is filtered out. Appendix I describes the design and construction of such a filter. The resulting SSSC wave is then boosted to final transmitter frequency in one or more stages by mixing, or hetrodyning with one or more oscillators of suitably higher frequency, each followed by an adequate filter to remove the unwanted new sideband. This latter filter is very simple, consisting of little more than a tuned amplifier stage, or a simple tuned circuit. The filter method of sideband elimination, then in reality consists of two or more oscillators and balanced modulators (or mixers) in addition to the main and secondary filters.

The second method of sideband elimination, by phasing, may be more readily understood if the vector diagrams of Figure 9 are first studied. From the equation of an A.M. wave (100% modulation used for convenience), it is seen that the final voltage consists of the same three components previously mentioned. As these are of different frequencies, namely ω_c, $(\omega_c + \omega_s)$, and $(\omega_c - \omega_s)$, any single vector diagram representing all these components should indicate the rotating frequency, and it should be remembered that the vector diagram represents only the instantaneous position of the vectors -- that their <u>relative</u> position will change from instant to instant. Now, for any random instant of time, the vectors may have the relative position as shown in Figure ((A).

The same position may be constructed as shown in Figure 9(B), by noting that with the sideband vectors now originating from the moving point, the end of the carrier vector, their frequency of rotation <u>about</u> the <u>moving point</u> will be their frequency of rotationabout the origin minus the frequency of rotation <u>of</u> the moving point. For the upper sideband, the frequency is $(\omega_c + \omega_s) - \omega_c$, or just ω_s. For the lower sideband, the frequency is $(\omega_c - \omega_s) - \omega_c = -\omega_s$. To obtain the value of e_o, it is necessary to vectorally add the components, as is done in Figure ((C).

For the position of peak value of e, the vector position is that of Figure 9(D). Let this be called the reference position. Suppose, now, that to a signal having the reference position, a signal, which at the same instant of time has a vector positionas shown in Figure 9(E), is added. It is further assumed that the respective lengths of vectors are identical. On adding, it is seen that the lower sideband vectors cancel each other, while the upper sideband vectors add. The carrier components also add in quadrature, of course, but this is not significant. The important point is that one sideband has been eliminated. The problem, then, resolves itself into the production of two amplitude modulated signals, e_o and e_o', that have the indicated phase differences. This may be accomplished in two steps. First, take a signal equal to e_o and

shift it through 90 degrees, as indicated by Figure 10(A). Next, shift the audio input by 90 degrees, which results in Figure 10(B) and is the required wave of Figure 9(E). The The block diagram of Figure 10(C) indicates a method of generating the SSSC output by this phasing method. The use of balanced modulators eliminates the carrier. Of course, all components must be accurately balanced for satisfactory operation. The audio phase shifter is the only item not of routine design. Appendix II is a published article[4] on this subject and presents a basis for design.



CHAPTER III

SSSC AND SSRC RECEPTION

Before considering reception of SSSC or SSRC, the subject of detection, or demodulation, of modulated waves in general should be discussed. Detection, as principally used, is of three types -- square-law, linear, and hetrodyne. The first two of these are closely related.

Referring again to Figure 6, it is seen that a typical characteristic curve of a triode (i_p vs. e_g) is made up of essentially straight section, followed by a knee, followed by another more-or-less straight section. The shape of this curve is also representative of the i_p vs. e_p characteristic curve of a diode. If a detector is operated principally in the region of the knee of the characteristic curve, it is called a square-law detector. The name is derived from the fact that this portion of the curve may be closely approximated by a second order equation. If, on the other hand, the voltage variations are very large, then the effect is to shrink the scale of Figure 6, so that the region of the knee shrinks in effect to a point. Thus Figure 6 could be represented by two intersecting straight lines. This type of detection is called linear

detection. Because linear detection necessarily involves large values of signal-voltages, the term "power detection" is sometimes applied. Conversely, the small values of signal-voltage used in square-law detection leads to the name of "weak-signal" detector. An arbitrary dividing point of about one volt is generally made.

In practice, a square-law detector usually employs a tube containing a control grid in order to obtain amplification in the stage as well as detection. The circuit of such a detector would be that of an amplifier except that voltages impressed are such as to cause the tube to operate principally in the region of the knee of the characteristic curve. Mathematical analysis of the detection process is rather simple. If e_o is the impressed modulated wave, and i_p is the detector plate current, for square-law detection,

$$i_p = a_1 e_o + a_2 e_o^2 \ .$$

For an A.M. wave,

$$e_o = E_c \cos \omega_c t + 1/2\ E_c M \cos(\omega_c + \omega_s)t$$
$$+ 1/2\ E_c M \cos(\omega_c - \omega_s)t.$$

Case I of Appendix III carries out the derivation of the output components. The balance of Appendix III repeats the procedure for different conditions of e_o. A summary of the results of Appendix III appear in Table I.

As was mentioned earlier, the results appearing in Table I indicate that reception of A.M. with only the carrier suppressed would be difficult in that, as shown in Cases II, III, and IV, the carrier must be reinserted at exactly the same frequency, and closely in phase with the original carrier, if the resulting signal is to be intelligible. Except with the most expensive equipment, this is difficult, or impossible to accomplish.

Mathematical analysis of linear detection is not so simple or straightforward as was that for square-law detection. Because of the discontinuity in the effective characteristic curves for linear detection, a Taylor power series may not be used to represent the entire curve. A convenient mathematical dodge may be used by considering only the audio components. These audio components will be contained in the equations of the upper and lower envelopes of the modulated wave. Appendix IV carries this method of analysis out first for a square-law detector to demonstrate the method, then for linear detection of an A.M. wave. Linear detection (and square-law detection also) of a SSSC wave in reality is the

CASE	TRANSMITTED WAVE	SETTING OF RECEIVER	AUDIO COMPONENTS OF OUTPUT OF DETECTOR	INTELLIGIBILITY
I	A.M.	----	$K_1 \cos \omega_s t$ $K_2 \cos 2\omega_s t$	Excellent if K_2 made small
II	Suppressed Carrier	Carrier at $\omega_c + \Delta$	$K_3 \cos(\omega_s + \Delta)t$ $K_3 \cos(\omega_s - \Delta)t$ $K_2 \cos 2\omega_s t$	Unintelli- gible
III	Suppressed Carrier	Carrier at $\omega_c t + \theta$	$K_3 \cos(\omega_s t + \theta)$ $K_3 \cos(\omega_s t - \theta)$ $K_2 \cos 2\omega_s t$	Excellent if K_2 and θ made small. Intensity may be reduced to zero if θ large.
IV	Suppressed Carrier	----	$K_2 \cos 2\omega_s t$	Unintelligible
V	[illegible]	----	----	No audible signal
VI	[illegible]	Carrier	$K_3 \cos \omega_s t$	Excellent
VII	[illegible]	Carrier at $\omega_c + \Delta$	$K_3 \cos(\omega_s - \Delta)t$	Excellent for $\Delta < 10$ Fair for $\Delta < 30$

$K_1 = a_2 E_c^2 V$; $K_2 = a_2 E_s^2 V^2/4$; $K_3 = a_2 C E_c E/2$

Table I. Square-law Detection of Various Modulated Waves (Summarized from Appendix III)

third type of demodulation -- hetrodyning -- and will be discussed in the next section.

Comparing the results of detection of an A.M. wave by square-law and linear detection, two conclusions of importance are apparent. First, square-law detection generates a second harmonic of the signal, which distorts the output signal to a degree equal to M/4, while no such component is produced by linear detection. Next, the magnitude of the signal frequency output (audio) of a square-law detector is proportional to the square of the carrier magnitude, while for a linear detector, it is proportional to the first power. These conclusions have resulted in the almost universal adoption of linear detection for conventional A.M. receivers because less distortion results, and the receiver is less affected by signal fading conditions. This situation may appear irrevelant at this time, but if SSSC reception is to be attempted with a conventional receiver, it becomes important, as will be shown later.

Hetrodyning, the third method mentioned of demodulation, is the process of combining a locally generated wave with the incoming wave in such a manner that a third wave is produced which either is the signal component, or contains the signal component in modulated form. This device is used

in superhetrodyne receivers to convert the incoming wave to a lower (fixed and predetermined) frequency. It is also used in communications receivers to supply an audible signal when CW (code) is being received. Appendix V, taken from "Theory and Applications of Electron Tubes" by Reich, is an excellent discussion of the nature and process of this type of detection. As indicated there, the simple mixing of two waves of different frequencies would result in a beat-frequency wave which had much of the appearance of a conventional A.M. wave, however, it is a wave that is varying in phase as well as in magnitude. The envelope, which is of the difference frequency, can be obtained in the form of useful power only by sending the two waves through a non-linear circuit element, such as a square-law or linear detector. (As has been implied, the required non-linearity in a linear detector is provided by the discontinuity in the characteristic curve.) This process, in effect, amounts to the process of modulation, not demodulation. Sidebands of the sum and difference frequencies are produced which is the same as the result of A.M.

It is apparent now, that demodulation of SSSC will always be of the hetrodyning type, regardless of whether

a linear or a square-law detector, or mixer, is used. The process is basically one of mixing the incoming wave with a locally generated wave to give the audio frequency output. For detection using a square-law mixer, Cases VI and VII of Appendix III present a derivation of the resulting output components. The difference frequency is the audio frequency signal. No second harmonic distortion signal is present. Referring again to Appendix IV, it is seen that for a linear mixer, the output components are infinite in number, containing an infinite number of audio distortion components. The amplitudes of these distortion components are functions of h^{n-1}, where h is the ratio of the amplitudes of the incoming signal to the locally generated signal, and n is the order of the narmonic. The resultant amount of distortion becomes acceptable at values of h below 2. The type of harmonic distortion described here will be referred to as "rectification distortion".

We are now able to consider practical reception of SSSC. A conventional A.M. communications receiver is equipped with a local oscillator (in addition to the one used in the superhetrodyne circuit) to be used in reception of CW signals, as was mentioned above. This is usually provided with an external frequency adjustment control, so that it

amounts to a local, variable frequency, beat-frequency oscillator (BFO). It's frequency range is from somewhat above, to somewhat below the incoming carrier frequency after conversion to the intermediate frequency in the superhetrodyne mixer circuit. Thus, this oscillator should be usable in the demodulation of SSSC. When a SSSC wave is tuned in, only a garbled gibberish will be heard. This signal must be carefully tuned to maximum strength with the BFO and the automatic volume control switched off. Now the BFO is switched on -- the tuning control is not touched again. The frequency adjustment control of the BFO is slowly adjusted until the gibberish becomes intelligible. If this does not occur, the gain of the R.F. stages should be reduced to as low a value as possible. In fact, normally, the volume control should be on maximum, and the R.F. gain adjusted to just enough gain to provide adequate volume. The reason for this is that practically all conventional receivers employ linear detection. When this linear detector is used as a linear mixer for SSSC demodulation, then as has already been shown, the value of h (ratio of amplitudes of incoming signal and local oscillator) must be below .2 if the resultant distortion is to be acceptable. A lower value would be still more

desirable. If, after reducing the R.F. gain to as low as value as possible, adjustment of the BFO control does not result in an intelligible signal, then either the BFO control does not permit a wide enough frequency variation, or the amplitude of the BFO is too small to reduce h to an acceptable value. If such is the case, then the receiver must be modified before SSSC transmissions may be received.

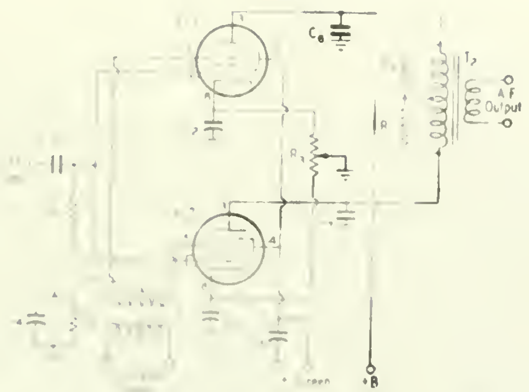
As indicated in Table I, the frequency of the local oscillator must be within less than 50 c.p.s. of the original carrier. It is desirable for it to be within less than 10 C.P.S. If music were being transmitted, the frequency difference would have to be less than 5 c.p.s. Thus it should be obvious that the local oscillator must have very good stability to prevent fading in and out of intelligibility. This degree of stability is provided in only the best of communications receivers, as it is not required for A.M. reception.

From what has been said above, it seems that the use of a conventional receiver for reception of SSSC, while possible with most such receivers, is at best, only a makeshift arrangement. This type of receiver is poor for two reasons -- the type of detector and the characteristics of the BFo. With a good quality communications receiver,

satisfactory SSSC reception is usually possible with only the addition of an external oscillator of very stable design and of sufficient amplitude.

In some cases, however, the high amplitude of the auxiliary oscillator overloads the detector, and thus introduces additional distortion. If, in addition, the detector is changed to a more suitable type, then reception is possible that, while not as good as that of the huge receivers employing multiple crystal filters and fixed frequency operation, is very good indeed. An excellent detector for this use is shown in Figure 11. Comparing Figure 11 with Figure 4 indicates that the demodulator is a form of the balanced modulator. As even for square-law mixers, some rectification distortion will occur, the use of a balanced mixer of the square-law type permits the cancelling out of the second harmonic distortion components, thus the advantages of both the linear and the square-law mixers are obtained with none of the disadvantages.

The problem of design of a local oscillator that is stable to within several c.p.s. is a difficult one. In addition, the additional complication of the tuning process due to having to adjust the frequency of the local oscillator for each station received can be serious if



[illegible] Balanced [illegible] converter for single-sideband [illegible]. [illegible] substitutes for the conventional [illegible] serving as a second detector [illegible]

16

[illegible]

[illegible] volts.

[illegible]

[illegible] center-tapped secondary [illegible]

[illegible] push-pull primary capable of [illegible]

rapid change of station is necessary. If the carrier were only reduced, say 20 db, rather than eliminated, then the carrier would be received. Practically all the advantages of SSSC transmission would still be realized, and the small amount of carrier received could be used to provide a synchronizing signal for the local oscillator. The advantages are obvious. The disadvantages is that reception of this transmission on a receiver not having a synchronizing device would be disturbed by a low-frequency beat signal formed between the carrier transmitted and the reinserted carrier. This disturbance would be of relatively small magnitude is the transmitted carrier were reduced as much as 20 db.

Several methods of synchronizing the local oscillator with the incoming carrier are in use. For the most complex receivers, the received carrier is separated out and used to drive a servomechanism that mechanically adjusts the oscillator variable condenser. This allows extremely good control but is also very expensive. This class of receiver will not be discussed in this paper. Another method of synchronization control is the use of a reactance tube, which is controlled by the D.C. component of the carrier out of the demodulator. This reactance tube, in turn, by providing a variable capacitive reactance shunted across the local oscillator tank circuit, tends to synchronize the local oscillator.

There is room for much development work in the field of relatively simple synchronizing devices.

One of the advantages of SSSC is that as half the frequency spectrum is required as compared with A.M., the ratio of signal to noise will be doubled in a receiver that is selective enough to accept only the usable sideband spectrum. This desirable feature may be realized from an A.M. transmission if the receiver is capable of removing one sideband. By using the phasing methods described under SSSC transmission, an adapter may be designed that will permit the removal of either sideband in a standard A.M. transmission. Into this converter can be incorporated the following features:

(a. Single sideband reception of A.M. transmissions

(b. Exalted carrier reception of A.M. transmissions

(c. SSSC reception (either sideband)

(d. Automatic frequency synchronization on either A.M. carrier, or SSRC transmissions.

Figure 12 is a block diagram of such an adapter. Consider first Figure 13, which is the enclosed portion of Figure 12, neglecting the low-pass R-C filter and the reactance tube. The input from the receiver I.F., which would have gone into the receiver detector, is fed into two identical demodulator tubes. A local oscillator, tuned to carrier frequency, feeds directly into one of the demodulator tubes, and through a 90°

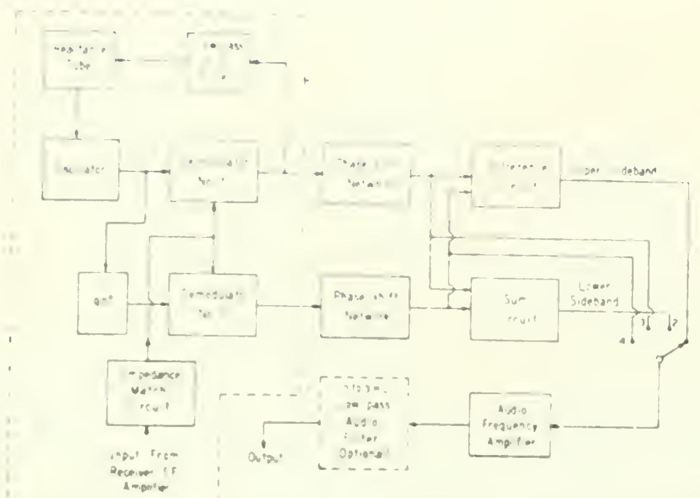


FIGURE 12. A Block Diagram of a Single-Sideband Receiving System.[15]

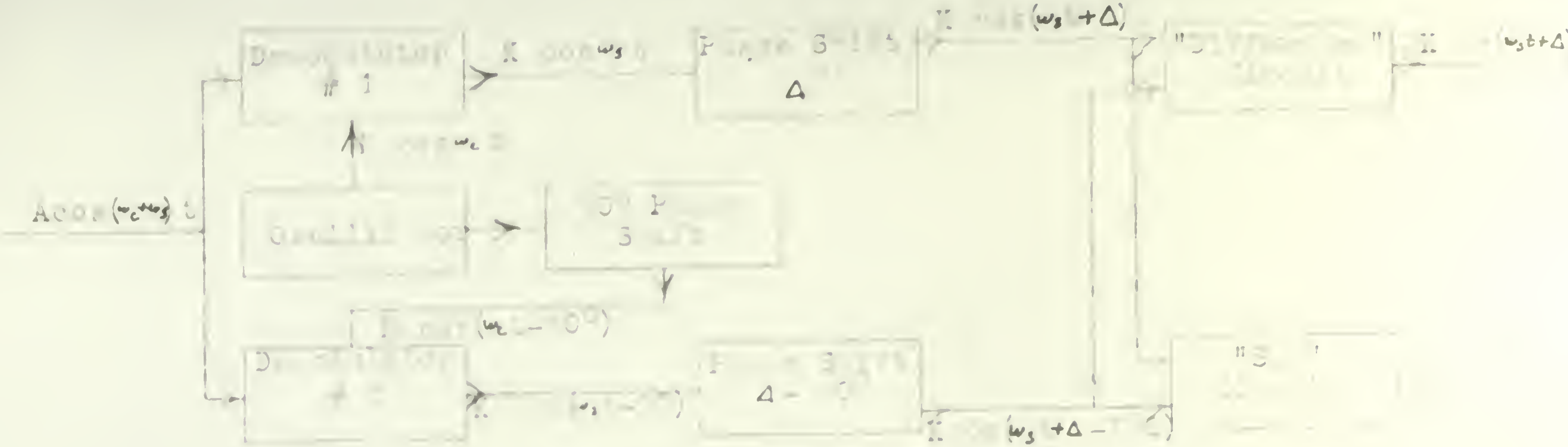


FIGURE 17(A). Separation of Sidebands by Phasing Method -- Upper Side Band Received.

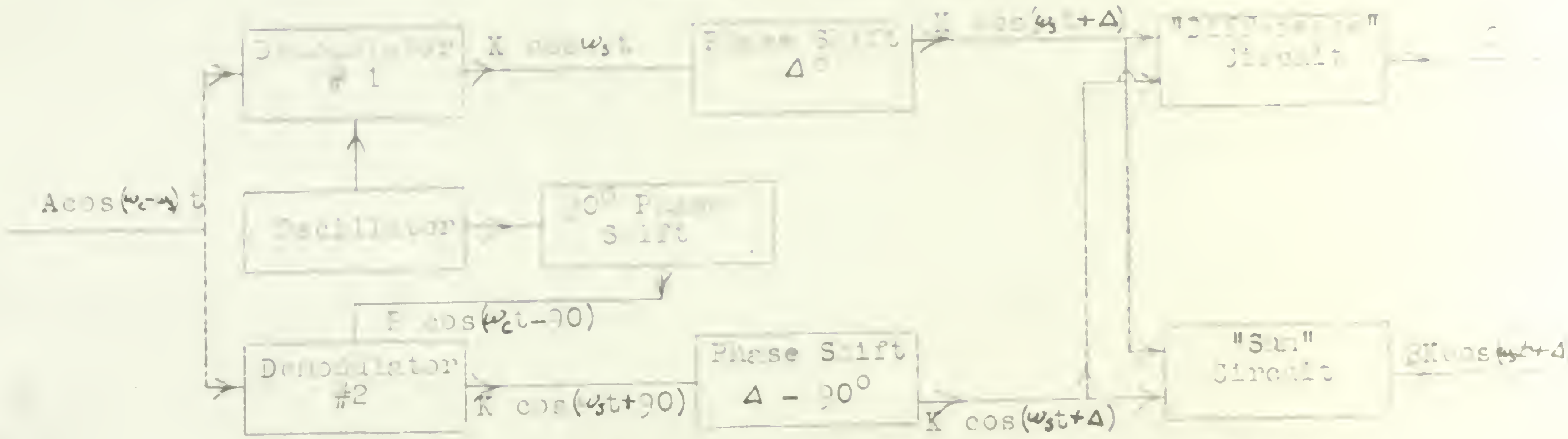


FIGURE 17(B). Separation of Sidebands by Phasing Method -- Lower Side Band Received.

phase-shifter, into the other. The amplitude of this local oscillator signal is approximately 100 volts, thus it controls the operation of each demodulator to give exalted-carrier demodulation of A.M. If an incoming signal is of somewhat different frequency than the local oscillator, then a beat note will be produced. Each demodulator will put out this beat note, let us say of 1000 cps. The frequency and amplitude will be the same for the two demodulators, <u>however</u>, there will be a 90° phase shift between the two. Further, this 90° phase shift will be either positive or negative, depending on whether the incoming signal is of higher or lower frequency than that of the local oscillator. These two audio signals leave the demodulators and pass through two phase-shift networks that together, produce a futher phase shift of 90° in a fixed direction. Let us assume that the incoming wave was 1000 cps. higher than the local oscillator frequency. This would result in two 1000 cps audio signals from the two demodulators that differ in phase by 90°. Now, these two audio signals are fed into the phase-shift networks that introduce an additional phase lag of 90°. The two audio signals thus emerge from the phase-shift networks 180° apart. If the two are added, zero audio signal results.

If the two are subtracted, then maximum signal will result.

Suppose now, that in addition to the incoming wave of 1000 cps greater frequency, a wave of 1000 cps less than the local oscillator is also fed into the demodulators. This is equivalent to an A.M. wave modulated at 1000 cps. Then the higher frequency will result in the same output as described above. The lower frequency, however, will result in audio signals out of the demodulators that lead each other by 90°. When these are passed through the phase shifters, the output signals will be exactly in phase. Thus, in a "sum" circuit, the 1000 cps note due to the lower incoming wave (lower sideband) alone would be present. In a "difference" circuit, the 1000 cps note due to the higher frequency wave (upper sideband) alone would be present. The two sidebands have been effectively split, and reception is possible on either alone. Of course, if only one sideband is being transmitted, then the audio signal will appear at either the "sum" or the "difference" circuit, but not at both.

The low-pass R-C filter permits only the D.C. component of the first demodulator to enter the reactance tube, which, as already mentioned, serves to synchronize the local oscillator with the incoming carrier.

In order to appreciate the advantage of single sideband reception of A.M. waves and the use of exalted carrier detection, some discussion should be given to the problems of

radio communications. This does not include commercial broadcasting, as in most cases, sufficient separation is provided either geographically or in transmission frequency to prevent interference. On short wave communications frequencies, however, geography is of little help, and the number of stations is very much greater. The frequency separation between stations may approach zero. The amateur channels are excellent examples of the situation being discussed. Very narrow bands are provided, and thousands of amateurs attempt to say their bit at the same time. The condition may be expected to be as bad on commercial communications channels in a few years. If a receiver is tuned to one station and another, removed in frequency by 5 KC (which is very common) is transmitting, then a 5 KC audio beat note will result in a conventional receiver. In actual practice, this is the most common type of interference present, not only on the amateur bands, but also on many of the communications channels. (If suppressed carrier transmission were employed, this disturbance would be reduced to a negligible value). The use of an adapter herein described would permit the receiver operator to choose the particular sideband that was least disturbed by these interfering hetrodynes. A further difficulty encountered is the blanketing effect of a strong transmitter on a relatively weak station if

the two are not far removed in frequency. The use of an exalted local carrier, accurately tuned to the weaker station frequency, would cause the detector to reject the strong station in favor of the desired weaker station. A further advantage of exalted carrier detection is also present. In detection of an A.M. wave as received, the value of M must not exceed unity or very great distortion will result. This is the same type of distortion that would be present if a transmitter were modulating greater than 100%. Even if the transmitter has an M of less than unity, frequently selective fading will reduce the relative magnitude of the carrier. This may be noticed at times when receiving a broadcast station that is some distance away, particularly at dusk, when a shift of the reflected ionized layer is occurring. The use of a strong locally inserted carrier greatly reduces difficulties of this type.

In the adapter described here, there are no inherent limitations on the width of the audio band as is the case for other highly selective circuits. However, for communications use, an audio frequency range of from approximately 200 cps to 2800 cps is adequate for satisfactory intelligence. If an audio filter is employed at the transmitter, appreciable savings in power result from the blocking of frequencies outside this range. At the receiving end, the effective selectivity of the

receiver may be greatly improved, and effectively a nearly ideal rectangular selectivity curve is obtained, as is shown in Figure 14. In Appendix VI are given the details of construction of an adapter utilizing the principles outlined above. In order to gain first-hand experience with construction and application of such a unit, this adapter was constructed. Operation is very satisfactory. A reduction in the unwanted sideband of 35 db was obtained.

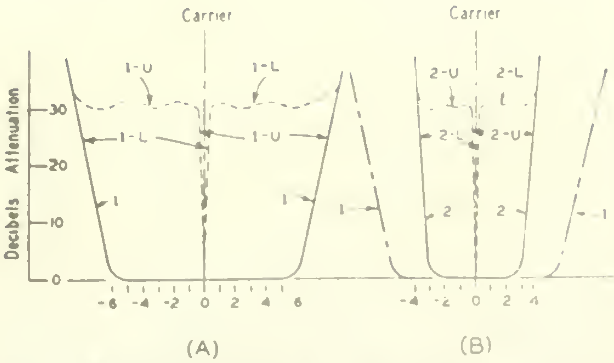


Fig. 14 Over the range for which the audio phase-shift networks hold close to 90° difference, the apparent i.f. response of the system is determined (in combination) by the i.f. and audio bandwidths. The characteristic with no audio filter is shown at (A), and (B) demonstrates how greater effective selectivity is obtained with an audio filter. 15

CHAPTER IV

ADVANTAGES, DISADVANTAGES AND LIMITATIONS OF

SSSC AND SSRC

Considerable confusion can arise in the consideration of advantages of SSSC transmission because of the variety of viewpoints possible. That very definite advantages do exist is not subject to dispute, but the degree of the advantages depends on what is desired of the transmitter. Figure 15 represents a conventional A.M. transmitter. Say for example,

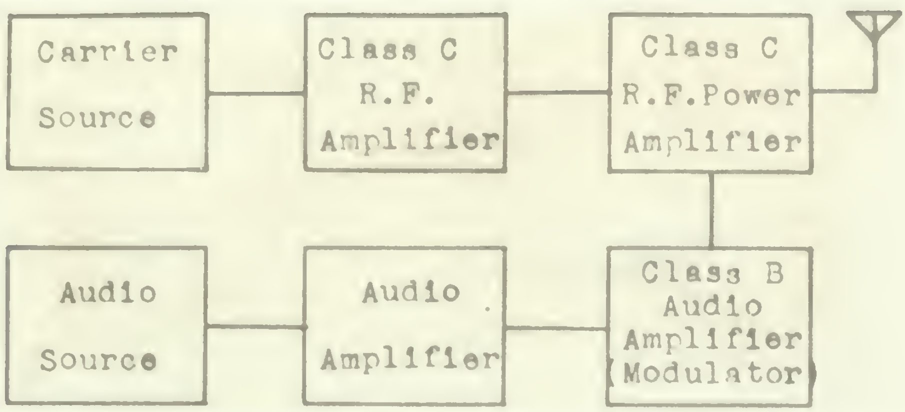


Figure 15. Block Diagram of a Conventional A.M. Transmission System.

it is rated at 10,000 watts. This is the power output of the R.F. power amplifier with no modulation. As the efficiency of a good Class C.R.F. amplifier is about 80%, the power input to the stage will be approximately the 12,500 watts. The required audio power for 100% modulation will be half

of the power input to the final R.F. amplifier, or 6250 watts. At 100% modulation, the average power radiated by the transmitter is .80(12,500 + 6250), or 15,000 watts, of which only 5000 watts is useful in the transmission of intelligence. Now, an often overlooked point is that for a normal human voice, the ratio of average to peak power is never larger than 1/2, and it is usually 1/4 or less. Then for a peak audio power of 5000 watts, the average is only 1250 watts. Thus for a 10,000 transmitter, the average power radiated is 11,250 watts, and the peak power is (four times the carrier power) 40,000 watts. Amplifier tubes are rated in size by their power dissipating ability. In this hypothetical transmitter, at no modulation, average dissipated power is 2500 watts. At 100% modulation on peaks of normal voice, the average dissipated power is 2810 watts, and the peaks of dissipated power is 10,000 watts. Thus the tube in the final R.F. amplifier would have to have a power rating of 2810 watts and would have to be able to withstand peaks of 10,000 watts.

From this point, two different paths may be followed. First, consider that it is only desired to transmit an average of 1250 watts, with a peak of 5000 watts of intelligence -- the same as for the original transmitter -- by SSSC transmission.

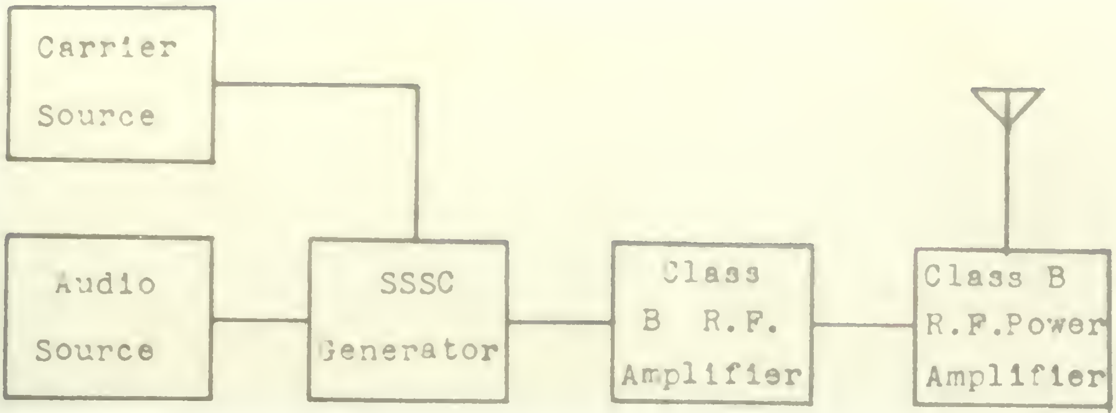


Figure 16. Block Diagram of SSSC Transmission System

For SSSC, this power represents the entire output. There is no carrier. Further, this peak of 5000 watts of intelligible power is the peak power of the transmitter, because for uniform intensity of audio signal, the envelope of a SSSC wave is a pair of horizontal lines. Note, however, that linear R.F. amplifiers must be used for SSSC, and the efficiency of a good Class B stage is about 70%. Thus, the average power input to the final R.F. amplifier is 1250/.70, or 1786 watts; and the dissipated power is 535 watts. The peak of power dissipation in the final amplifier is 2140 watts.

In Figure 16, it is important to note that all stages up to the first R.F. amplifier are of negligible power. Receiver-type elements are employed. The only power stages are the R.F. amplifiers. No audio power amplifiers are required. Comparing Figures 15 and 16 in this light, it may be estimated that roughly 25 KW of line power is required for the A.M. transmitter, while approximately 8 KW of line power is required by the SSSC transmitter.

For a SSSC transmitter designed to transmit an equal amount of intelligible power, the advantages may be summarized as follows:

(a. Savings of approximately 80% in size and cost of final amplifier.

(b. Savings in cost, and reduction in size of transmitter, by complete elimination of audio power amplifiers and power modulators.

(c. Savings of approximately 70% in cost of operating power.

No significant disadvantages or limitations have yet been introduced.

Suppose, now, that the original A.M. transmitter is on hand, and it is desired to realize all possible output power advantages by conversion to SSSC. What, then,

would be the benefits to be derived. Reviewing, the A.M. transmitter contains a Class C final amplifier, capable of dissipating an average of 2812.5 watts, with a dissipation peak of 10,000 watts. Operated as a Class B, SSSC final amplifier, this would be able to deliver 9370 watts, average, of intelligible power, with peaks of 33,333 watts. As this ratio is greater than the estimated 1 to 4, using the peak power as the limiting criterion, an average power of 8320 watts could be transmitted. As before, the audio power amplifiers are eliminated, and operating power consumption is reduced to approximately 13 KW.

Summarizing the advantages to be obtained by converting an existing A.M. transmitter to SSSC to obtain greatest possible output power, they are:

(a. Intelligible power transmitted increased by approximately 650%.

(b. Audio power amplifiers eliminated.

(c. Savings of approximately 50% in cost of operating power.

Again, other than the work of converting an existing transmitter, no appreciable disadvantages or limitations exist.

Before leaving the transmission end of SSSC, mention should be made of several practical items that require attention. First, the power supply to all stages of a SSSC transmitter must be extremely well filtered to prevent spurous radiations and prevent undue distortion of the audio signal. Next, all stages must be carefully designed and operated, so as to be as nearly linear as possible, otherwise, the magnitude of the suppressed sideband will become appreciable due to the generation of an A.M. signal in the non-linear elements. In general, more care and attention to details of design is required in a SSSC transmitter if really satisfactory operation is to be obtained. These items are problems of design, construction and operation and should not be considered as disadvantages or limitations of the system.

Advantages and limitations inherent in single sideband reception of either A.M. or SSSC transmissions have been adequately discussed. The limitations and most of the disadvantages of theSSSC system lie in the reception end. Summarizing these from the discussion of SSSC and SSRC reception, they are:

Disadvantages--

(a. Reception impossible on all but communications-type receiver.

(b. Reception procedure more complex, and in some cases, impossible on conventional communications receivers. Results are only fair.

(c. Even with moderately priced adaptors, the problem of tuning is more complex than for A.M. Simple automatic lock-in circuits are not thoroughly developed (assuming a reduced carrier is transmitted).

(d. Everpresent problem of satisfactory local oscillator stability is difficult to satisfy without either loss of flexibility of operation or high cost.

Advantages --

(a. Increase in signal-to-noise ratio.

(b. Ability to choose sideband having least interfering signals.

(c. Ability to better separate desired weak signal from undesirable strong signal.

Some of the advantages of the SSSC system can not be assigned to either the transmitter or the receiver alone. Of greatest importance in this classification is the fact that only half the R.F. spectrum is required for SSSC as compared with A.M. This permits either a wider separation of stations, or a larger number of simultaneously operating

stations. The second, is a result on reception of suppressing carrier. As mentioned before, on communications channels, one of the greatest sources of interference is the productions of very intense hetrodyne notes as a result of mixing of carriers not far removed in frequency. Elimination of carriers would, of course, completely eliminate this disturbance. Further, as no carrier is transmitted, if the transmitter is operating properly, when the operator is not speaking, no signal at all is transmitted (except possibly a greatly reduced carrier, which would not effect the results.) This immediately permits breakin operation with no special equipment, and on the same frequency.

In view of the above discussion, several conclusions seem reasonable. First, use of SSSC on commercial broadcast bands, while desirable from the point of view of station economy and efficiency of frequency utilization, is not feasible because of the handicaps of reception. Not only would converters be required for the 80 million home receivers now in use, but also, an educational program in tuning procedure would be required. This is ridiculous, of course.

For commercial communications operation, however, the story is entirely different. Except for a few classes of communications, the use of A.M. is not only extremely

wasteful of equipment and power, but the waste of 50% of the available frequency is hardly excusable with SSSC in it's present state of development. The number of communications stations is increasing rapidly with time. For a long while, the demand for additional frequency space was met (largely due to the pioneering of the amateur radio operators) by going to higher and higher frequencies. With the spectrum of communications circuits approaching the frequencies of light, the limit seems to be in sight. Further, the communications circuits for which SSSC is most applicable are not satisfactory at frequencies much above 30 megacycles. The available space, then, seems to be from 50 to 550 KC and from 1.7 to 30 MC.

The expense of conversion of a transmitter would not be prohibitive. Even this cost could be more than regained by savings in operating power costs over a reasonable period of time. To the writer, it seems only reasonable to expect that more and more newly constructed communications stations will be of the SSSC variety, and that conversion of existing transmitters will accelerate as operating procedure knowledge is more widespread. It does not seem unreasonable to expect that in the not too distant future, the great majority of communications links will employ SSSC systems. For amateur operation, the present situation

in the "phone" bands is a good example of what is to be expected in commercial channels as the number of stations increases -- unless SSSC is more universally adopted. Perhaps if SSSC comes into widespread use in the amateur channels, not only will the bad situation be cleared up within those channels, but as has happened in the past, developments of technique and simplifications of equipment will cause a more rapid spread of SSSC into the communications channels.

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APPENDIX I. (From QST for June 1949.)

A Filter Design for the Single-Sideband Transmitter

BY FRED M. BERRY,* W0MNN

S[illegible]

[illegible]

Briefly, [illegible] "phasing" method [illegible] sideband by combining two balanced modulators with 90-degree [illegible] networks.[2] This system enables the sideband to be produced directly at the desired output frequency. While this has certain advantages, there is little assurance that the necessary high degree of phase and amplitude balance will be maintained over long periods of time. There is also difficulty in determining with simple test equipment whether the [illegible] sideband [illegible] and [illegible] suppressed.

[illegible] method of generating single-sideband signals, a double sideband is first generated in a balanced modulator where the carrier is eliminated, and the filter removes the undesired sideband by "brute force." While this method does not have the "finesse" claimed for the phasing method, it is positive, and requires no critical adjustments. Since the filter is a passive network, sideband suppression is not affected by [illegible] variations, tube gains, etc.

Filters using only inductors and capacitors are practical only at frequencies of the order of 10 to 50 kc., and the sideband must be obtained at some low frequency and heterodyned to the desired output frequency. This is not a serious handicap, and enables the output frequency to be varied without disturbing the sideband-generating portion of the circuit. Contrary to the statement made by some, the selection of either upper or lower sideband is simple, requiring only a frequency change of one of the oscillators.

The block diagram of a practical single-sideband transmitter is shown in Fig. 1. The selection of upper or lower sideband is accomplished by switching the frequency of the second oscillator. In the notation of Fig. 1, "USB" and "LSB" indicate the position of sideband at the points noted with respect to the input speech frequencies. This is not to be confused with the particular sideband of the second oscillator that is selected by the second i.f. Careful study of Fig. 1 will make this clear.

Although not directly indicated in Fig. 1, the

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[1] Goodman, "What Is Single-Sideband Telephony?" QST, Jan. 1948; Norgaard, "What About Single-Sideband?" QST, May 1948.

[2] Norgaard, "A New Approach to Single Sideband," QST, June 1948.

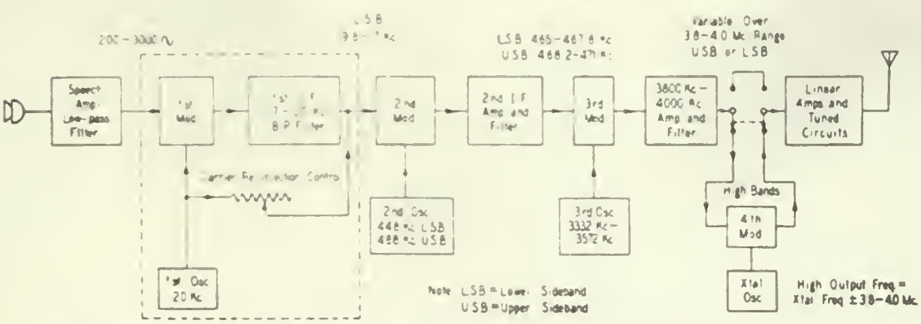


Fig. 1 — Block diagram of a typical single-sideband suppressed-carrier transmitter or exciter. The equipment inside the dashed area is described in this article.

[illegible] and the series [illegible] from [illegible] high-frequency side [illegible]. When the [illegible] combined, the inductances and capacitances [illegible] may be combined [illegible] to give the [illegible] for the complete filter.

The input and output impedance characteristics are the same as those of the mid-shunt constant-k type of filter of the same cut-off frequency. This sort of termination impedance is [illegible] and works well either directly from a ring modulator or resistance terminations.

The design impedance R of 1000 ohms was selected to give desirable component values and desirable input and output impedances obtained by transformer action in the end inductors.

The resultant values calculated from the design formulas of Fig. 2 for each section are given in Fig. 3 and in Fig. 4 for the two sections combined.

To those who may wish to calculate similar filters, note that if sections are to be joined, the design impedance and the cut-off frequencies must be the same for both sections, although the frequencies of infinite attenuation may be different.

The filter of Fig. 4 might now be constructed and if proper components are available, the insertion loss between 1000-ohm resistive impedances would be approximately that of Fig. 5. A low dissipation factor (high Q) is necessary in most of the components to obtain the required characteristics. Resistive losses internal to the filter not only will cause a greater loss at all frequencies but will "round off" the edges and prevent the rapid rise of attenuation needed just outside the passband.

Except in the case of C_1 and C_7 (Fig. 4) mica or other low-loss types of capacitors are necessary for proper filter action. C_1, C_4 and C_7 are large values for mica capacitors and would be expensive. Ordinarily they would have to be made from a large number of paralleled units. C_1 and C_7 appear directly in parallel with the terminating resistances and it has been found that good-quality paper capacitors are satisfactory here.

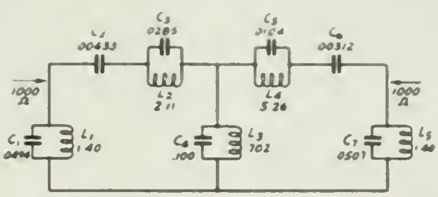


Fig. 4 — Component values of the filter after combination of the parallel components. Values are in µfd. and mh.

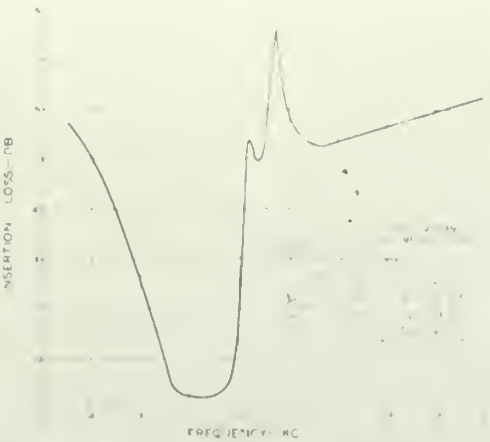


Fig. 5 — Attenuation characteristic of the filter shown in Fig. 4. The test set-up diagram refers to Fig. 6.

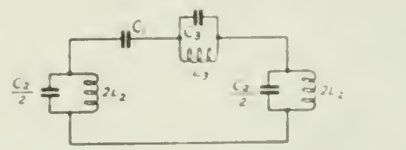
C_4 is located internally and must be of low-loss type for best results. It is possible, however, to use impedance transformation at L_3C_4 and permit a smaller mica capacitor to be used for C_4 at the expense of a larger value for L_3. The method of impedance transformation employed also permits a relaxation of the capacitor tolerance. Any reasonable value may be used for C_4 provided the inductor is adjusted to the correct antiresonant frequency. The correct impedance may then be regained by tapping L_3. With inductors of high coupling between turns, the proper point of tapping is such that the inductance between common and tap is approximately that of the value of L_3 before transformation. The modification with the more desirable values is shown in Fig. 6.

Component Tolerances

In the filter of Fig. 4, the tolerance of some of the elements is quite critical, particularly that of the series arms. It has been found in the design of filters of this type that the tolerance of LC ratios is not particularly critical provided the correct resonant and antiresonant frequencies are maintained. Practically, this leads to the selection of capacitors to a tolerance of ±5 per cent or better, and resonating each LC circuit to the correct frequency by turn adjustment of the inductor. The maximum possible error of 10 per cent in the impedance match between junctions of the filter arms is not serious. Greater tolerances will cause a "ripple" in the passband and other deviations from the desired characteristics. In following this procedure, note that the series arms of the filter have both a resonant and an antiresonant frequency, with the inductor as a common element for both. Obviously, the inductor could not be adjusted

[illegible] 28 [illegible] 17–20 kc. [illegible]

The third i.f. requirements are similar to those of the second i.f. except that, with the frequencies chosen, the selectivity requirements are more lenient. It would be quite practical to employ a mixer-modulator, or one of the balanced type, for the third modulator. Further notes on the complete exciter or transmitter are beyond the scope of the present article, as the first i.f. filter is considered to be more difficult to the average constructor and is deserving of full consideration.[1,2]



$C_{1k} = \frac{f_2 - f_1}{4\pi f_1 f_2 R}$, $L_{2k} = \frac{(f_2 - f_1)R}{4\pi f_1 f_2}$ $k = \sqrt{\left(1 - \frac{f_1^2}{f_{2\infty}^2}\right)\left(1 - \frac{f_2^2}{f_{2\infty}^2}\right)}$

$C_1 = a C_{2k}$, $C_2 = m C_{2k}$

$C_3 = \frac{1 - m^2}{4m} C_{2k}$ $a = \frac{(1 - m^2) f_{2\infty}^2}{4 f_1 f_2}$

$L_2 = L_{2k}$

$L_3 = \frac{m}{a} L_{2k}$ $m_1 = \frac{f_1 f_2}{f_{2\infty}^2} + k$

Fig. 2 — The basic *m*-derived pi section used in the filter.

The primary purpose of this article is to describe in detail the construction of a highly-selective first i.f. filter that can be built at a reasonable cost and with a minimum of special test equipment.

[1] The general technique is shown by Nichols, "A Single-Sideband Transmitter for Amateur Operation," *QST*, Jan. 1948.

[2] In the original manuscript Mr. Berry showed the 2nd oscillator Fig. 1 on either 448 or 485 kc. It was changed as shown because this maintains the output suppressed carrier on the same frequency when transmitting either upper or lower sideband and it is a little easier.

Filter Design

[illegible]

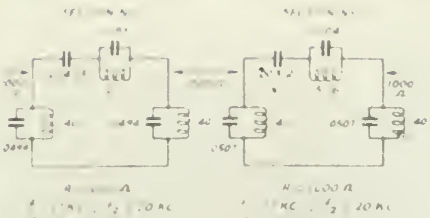


Fig. 3 — Component values of the individual pi sections of the filter. Values are in μfd. and mh.

[illegible] number of inductors. This filter is designed for selection of the lower sideband, but since the position of the sideband may be altered at the output of the transmitter in a succeeding modulator stage, this is no handicap.

A figure of approximately 40 db. reduction of the undesired sideband was selected as a practical value. It is believed that values much lower may tend to limit operation on the adjacent channel (when sufficiently selective receivers are in use). Values much over 40 db. would probably not be worth while even if a greater ratio were obtained at the output of the filter, because intermodulation in succeeding stages of the transmitter is likely to introduce spurious beat products of low intensity. (Note: In any single-sideband transmitter, improper amplifier bias and overloading in the linear amplifiers is to be avoided as the effect is similar to an overmodulated a.m. transmitter, with its resultant splatter.[3])

A bandwidth of 2800 to 3000 cycles has proved satisfactory for commercial communication and is thought to be a practical one for amateur use.

A frequency band of 17 to 20 kc. was chosen in preference to one of lower frequency to reduce the selectivity requirements of the second i.f. filter, as previously noted. This rather high frequency (for a single-sideband filter) also has the feature of lower component values, lowering cost and making hand winding of the inductors feasible.

The filter consists of two *m*-derived pi sections of the type shown in Fig. 2. This type of section has one frequency of infinite rejection on the high-frequency side of the passband. By using two sections, one with the rejection frequency at

[3] [illegible] "Linear R.F. Amplifiers," *QST*, May, 1949.

independently [illegible] this desired independent adjustment [illegible] inductor arrangement is used.

Considering the [illegible] selected with a tolerance [illegible] be larger than the [illegible] may then be adjusted [illegible] to the correct [illegible] resonant [illegible] have fewer turns than the original calculated value of L_2. Leaving [illegible] connected across the exact number of turns necessary for [illegible] resonance, turns may be added to L_2 until the [illegible] combination of [illegible] in series with [illegible] and the added winding of L_2 [illegible] the correct frequency. [illegible] the impedance [illegible] per cent is permissible.

[illegible]

[illegible] of separate windings [illegible] and C_6 [illegible] operation [illegible] input and output [illegible] between [illegible] using [illegible]

In the design given, an impedance of 500 [illegible] was selected [illegible] winding of [illegible] copper-oxide [illegible] operates quite satisfactorily [illegible] impedance. The impedance [illegible] the output winding of L_6 is a compromise between desired voltage step-up and keeping the number of turns to a value that permits hand winding. The completed filter design [illegible] modifications is shown in Fig. 6. In the event other input or output impedances are desired, the number of turns and method of connection of these added windings may be altered to meet the requirements. Since the impedance varies directly with the inductance of the windings (with 1.4 mh. the inductance for 1000 ohms impedance), the required inductance in millihenrys for any new impedances may be found by dividing the new impedance in ohms by 1000 and multiplying the result by 1.4. The number of turns required can be found from the formulas for the inductors given in Fig. 6.

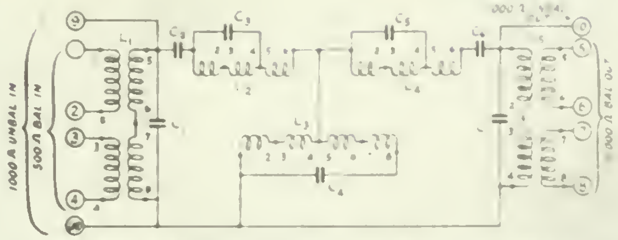


Fig. 6 — Revised filter of Fig. 4, with provision for balanced or unbalanced input and output.

C_1 — 0.049 μfd. ±5%, paper
C_2 — 0.0043 μfd. ±5%, mica
C_3 — 0.03 μfd. ± tol., mica
C_4 — 0.03 μfd. ±20%, mica
C_5 — 0.011 μfd. ± tol., mica
C_6 — 0.0031 μfd. ±5%, mica
C_7 — 0.051 μfd. ±5%, paper
L_1 (1–4) — 0.7 mh., 33 turns No. 26, bifilar
L_1 (5–8) — 1.4 mh., approx. 47 turns No. 26, bifilar
L_2 (1–4) — 2.0 mh., approx. 80 turns No. 26, bifilar.
L_2 (5–6) — 0 to 20 turns No. 26, single
L_3 (1–4) — 0.7 mh., approx. 47 turns No. 26, bifilar
L_3 (1–8) — 2.3 mh., approx. 86 turns No. 26, bifilar
L_4 (1–4) — 5.0 mh., approx. 125 turns No. 26, bifilar
L_4 (5–6) — 0 to 20 turns No. 26, single
L_5 (5–8) — 14.0 mh., 200 turns No. 28, bifilar
L_5 (1–4) — 1.4 mh., approx. 47 turns No. 26, bifilar

All wire Formvar or d.s.c. — see text

L_1 and L_5 wound on Western Electric P476930 core ring.

L_2, L_3 and L_4 on Western Electric P281395 core ring.

$$\text{Approx. turns P476930} = 1000\sqrt{\frac{161}{L}}$$

$$\text{Approx. turns P281395} = 1000\sqrt{\frac{79}{L}}$$

Resonant Frequencies

Inductor	*Capacity*	*Freq., kc.*
L_1 (5–8)	[illegible]	19.1
L_2 (1–4)	[illegible]	20.7
L_2 (1–6) with C_3 connected	C_2	19.1
L_3 (1–4)	0.1 μfd ±5%	19.0
L_3 (1–8)	C_4	19.0
L_4 (1–4)	C_5	21.5
L_4 (1–6) with C_5 connected	C_6	18.9
L_5 (1–4)	C_7	18.9

Filter Alignment

As has previously been mentioned, the LC combinations must be resonant at the desired frequency. In an m-type filter with closely-spaced rejection frequencies, it is very important to hold to very close frequency tolerances; while a constant error is not serious the spacing of one frequency to the next is critical.

Heretofore, it has been considered necessary to use expensive laboratory equipment, which is out of the reach of many. Signal generators for the range of 15 to 30 kc. are not common, and those available are usually not of sufficient accuracy. However, with the aid of a BC-221 frequency meter the main obstacle has been removed. The fundamental frequency range of the low band of the BC-221 is 125–250 kc., and it has sufficient output voltage to give a reasonable indication on most oscilloscopes. The BC-221 is used only as a calibration means for the test signal generator. The test generator may easily be made from the junk box, and the usual calibration problem

[illegible]

A method of measuring coil [illegible] [illegible] With the LC combination connected as [illegible] [illegible] the amplitude [illegible] when the frequency [illegible] the exact [illegible] resistor R of Fig. 7 should be the smallest value that will still give a readable indication. [illegible] as great but will be very sharp.

Coil Construction

In selecting inductors for the filter, the Q is of primary importance. Q values of at least 150 are necessary for all inductors except possibly [illegible] in parallel with the terminations, and losses here have much less effect. While many types of inductors might be used, the toroidal type has many advantages and core rings of molybdenum Permalloy are now available to the amateur.

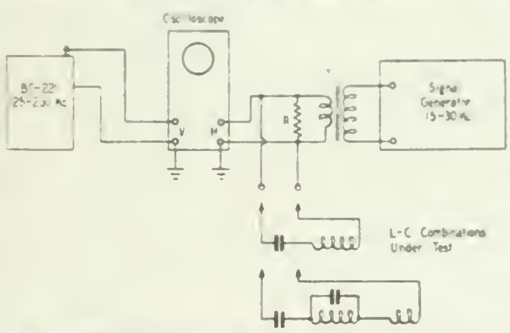
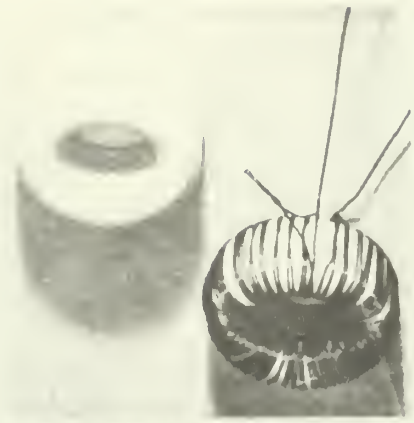


Fig. 7 — The method used for checking coil-and-condenser combinations. An accurate frequency check is obtained by using a BC-221 to check the 10th harmonic of the test signal generator. The LC combination under test is adjusted for minimum horizontal amplitude at the desired frequency.
R — 1 to 10 ohms, ½ watt. See text.
T — Step-down transformer. A 500-to-6-ohm audio transformer is suitable with generator outputs of 500 ohms or less.



A [illegible] toroid [illegible] of the type used in the [illegible] filter. [illegible] core is shown at the left.

Toroidal coils of this material are small in size and have a very low external field, and the inductance remains quite constant with power level and temperature. The coupling between turns is high, so that leakage reactance may be neglected in the design of the built-in transformers and tapped coils. The one disadvantage of using toroidal coils is the difficulty of winding, since the wire must be threaded through the core. However, in this filter special attention was given to keeping the inductances low, and winding is not too difficult. Two different grades of core material were used in the inductors for the filter of Fig. 6 (attenuation characteristics shown in Fig. 5).

Inductors L_2, L_3 and L_4 use cores having an effective permeability of 60, producing Qs of 200 to 250 at 20 kc. L_1 and L_5 cores were of 125 permeability, reducing the required number of turns and still permitting Qs of over 100. The construction data in Fig. 6 give the approximate number of turns of the inductors when using Western Electric core rings P476930 for L_1 and L_5, and P284395 for L_2, L_3 and L_4. P476930 and P284395 have nominal inductances of 164 and 79 millihenrys respectively per 1000 turns. The approximate number of turns for a specified inductance, as given by the manufacturer, is found by the formulas given in Fig. 6.

Since a tolerance is allowed on the capacitors, and the permeability of the cores varies slightly, the exact number of turns will vary and must be determined by measurement. For this reason sufficient length of wire should be allowed for the windings so that the additional number of turns necessary may be found by test. The extra length of leads will not

affect the [illegible] to proper length [illegible] turns has been determined.

Wire size is not [illegible] that given in Fig. [illegible] does not limit [illegible] Formvar [illegible] alent, is recommended [illegible] single silk or nylon [illegible] not of the Formvar type [illegible] of the possibility of shorted turns.

In order to reduce the [illegible] wire must be threaded through the [illegible] ring all windings are bifilar except the adjustment windings 5-6 of L_2 and L_4. In the bifilar type of winding, two wires are held together and wound as one. After winding, the start of one wire [illegible] may be connected to the finish of the other [illegible], thus connecting the two in the series-aiding manner. As in telephone practice, the numbering of

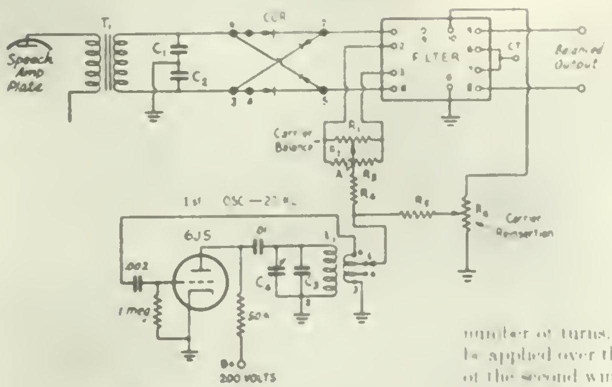


Fig. 8 — A suggested circuit for the 20-kc. oscillator and balanced modulator to be used with the filter.

C_1, C_2 — 0.05-μfd. ±20%, matched to within 1% by trial.
C_3 — 0.01-μfd. silver mica.
C_4 — 200-μμfd. variable or adjustable.
R_1 — 5000-ohm potentiometer.
R_2, R_3 — 150 ohms, ½ watt.
R_4 — See text.
R_5 — 3000 to 5000 ohms.
R_6 — 1000-ohm potentiometer.
L_1 — 284 turns or 142 bifilar, No. 26 Formvar or s.s.c., for coil 1-2. Coil 3-6 is 40 turns of same tapped every 10 turns. Both coils are wound on the same W.E. P284395 core ring.
COR — Copper-oxide modulator (Varistor). See text.
T_1 — Output transformer, plate to 500 ohms.

windings connected for series aiding is such that the direction of current at one instant is from *1* to *2*; *3* to *4*; *5* to *6*; etc. Thus if *2* and *3* and *4* and *5* are connected together and external connections are made to *1* and *6*, the windings are series-aiding.

In winding, the length of wire to be pulled through for each turn may be halved by starting at the center of the bifilar (doubled) length of [illegible] through the [illegible] end of the bifilar wire. The wire [illegible] evenly distributed around the [illegible] ring [illegible] possible.

The following procedure [illegible] and labeling of bifilar winding may be used. [illegible] ends of the completed winding [illegible] label them 1 and 3. Now by use of [illegible] locate the wire at the opposite end of the winding which displays continuity to 1. [illegible] will be 2 and [illegible] 3. With the exception of the input and output winding of L_1 and L_5, the free wires, 1 and 4, may be left long and any additional turns necessary may be obtained by winding on singly, with care that the wire continues through the core in the same direction.

The bifilar windings 5-6 and 7-8 follow the same procedure. However, when two windings are to be series-connected such as 1-2, 3-4, and 5-6, 7-8, care must be taken in selection of the end of winding to label 5, 7. The proper labeling is such that the wire ends 5, 7 pass through the core center in the same direction as the wire ends *1*, *3*.

The input windings of L_1 (*1-4*) and output windings of L_5 (*5-8*) are not critical in inductance and may be wound first to the specified number of turns. If desired, a layer of tape may be applied over these windings before application of the second windings.

L_3 (*1-4*) is wound and resonated with a 0.1-μfd. test capacitor to 19.0 kc. Adjust to the nearest turn that produces resonance closest to the exact frequency. C_1 and C_7 may be paralleled and used temporarily for the test capacitor. The second winding of L_3 (*5-8*) is now applied and series-connected with the inner winding, *1-4*. Turns are adjusted to secure resonance with C_4 at 19 kc. No connection is made to the tap during adjustment. Note that wide tolerances on C_4 are allowed and the exact number of turns of L_3 will depend on this tolerance.

L_1 (*5-8*) and L_5 (*1-4*) are wound and resonated with their associated capacitors, C_1 and C_7.

L_2 (*1-4*) is now wound and resonated with C_3. As previously mentioned, the value of C_3 may vary over a wide range (plus tolerance), and will determine the number of turns of windings *1-4*. Note that the total number of turns for L_2, including adjustment winding *5-6* depends only on the exact value of C_2. Thus if C_3 is large, winding *1-4* will have fewer turns, and *5-6* will have more. After resonating C_3 connect it in parallel with windings *1-4* and the combination in series with

[illegible]

The filter pass may be [illegible] using [illegible]. [illegible] may be used [illegible] filter is flat through the passband [illegible] have been tuned. If the oscillator is [illegible] band, the deflection [illegible] about 20 [illegible] should be [illegible].

The [illegible] of the components [illegible] the builder, but it is to be noted that the inductors may be mounted very close together and near metal surfaces without harmful effect, with the possible exception that L_1 and L_2 should be given some separation from one another. A metal screw may be used through the center of an inductor without harm provided it does not constitute a shorted turn, as it would if metal washers were used on both sides and the washers connected together.

A suggested schematic using the filter is shown in Fig. 8. The speech amplifier should have low- and high-frequency cut-off as with any phone transmitter. Some high-frequency attenuation may be obtained by the action of the secondary of T_1 with C_1, C_2. It is well first to run a frequency-response check on the speech amplifier including T_1, C_1, C_2 with the modulator disconnected and a 500-ohm resistor substituted.

The 20-kc. oscillator shown uses a toroidal inductor. Other types of oscillators will perform satisfactorily if the output impedance is held low. The number of turns of inductor L_1 and value of C_3 may be adjusted for proper frequency using the BC-221 and the proper feed-back adjusted by the secondary winding 3, 4, 5, 6. The taps on this winding are desirable to adjust the voltage at the junction of R_2, R_3 from 2 to 5 volts. Selection of 100 to 500 ohms for R_4 also permits some adjustment. R_5 should be as high as possible for least loading on the oscillator and filter, still permitting enough 20-kc. output for any desired amount of carrier reinjection.

One point not obvious is that R_2 and R_3 with R_1 in parallel are actually in series with the input to the filter. The values chosen normally [illegible] when a [illegible] varied from 300 to 3000 [illegible] speech amplifier [illegible] output from the [illegible] with a [illegible] oscilloscope should [illegible] the filter [illegible] matched [illegible] available in surplus [illegible] with the values given on Western Electric D162258, D163139 and D163[illegible]. The terminal numbering given for [illegible] Fig. 8 [illegible] these types.

Modulator balance for maximum carrier reduction is normally quite simple. A sensitive [illegible] or oscilloscope is connected to output terminal [illegible]. With no input to the speech amplifier and R_6 turned for minimum carrier, adjust the carrier balance control R_1 for minimum output.

Balance should be obtained near the center of the adjustment range. If not, a trimming resistor may be paralleled with R_2 or R_3. Some capacity unbalance in the Varistor or input winding of the filter may prevent sufficient carrier balance and small values of capacity may be added from filter terminal 1 or 2 to ground. Capacity may also be tried across C_1 or C_2.

Note that any hum in the speech amplifier will appear as an output carrier, but of course will be 60 or 120 cycles from the true carrier. Hum may be identified by temporarily shorting the primary of T_1.

Audio is now connected to the input of the speech amplifier and the level adjusted to a maximum of 0.25 volt at the output of T_1.

If the output of the speech amplifier is a pure tone the output of the filter should be a single frequency of 20 kc. minus the audio frequency. Using a sweep rate that is a submultiple of the audio input frequency, a check may be made for the presence of a modulation envelope. Such a trace represents more than one frequency in the output and may be caused by distortion in the speech amplifier or overloading of the modulator. A slight modulation pattern is permissible as this represents only a slight distortion of speech and not spurious signals out of the passband.

The modulator is now ready to be connected to the succeeding stages of the exciter.

CHARACTERISTICS OF SIDEBAND FILTER AS ACTUALLY CONSTRUCTED

The filter, for which the characteristics are shown on the next page, was constructed to the specifications in the article on filter design, with the exception that inductances of a much lower "Q" were substituted. The results of this substitution is readily apparent by comparing the solid curve below, with the corresponding characteristic curve of the designed filter. The attenuation peak at 20.5 KC is suppressed, and the attenuation in general is less over the suppressed sideband range. The most important effect, however, is the lowering of sharpness of cut-off from 20 to 21 KC.

In order that this filter be usable, the attenuation curve was shifted somewhat, as shown by the dashed curve, by shunting a .0017 mfd mica condenser across C_4.

The indication of an apparently negative attenuation in the bandpass region is due to a slight miss-match of impedance in the test circuit.

APPENDIX II. ... (RADIO) PHASE SHIFT NETWORKS
(ELECTRONICS, D... 1946.)

Wideband

Design criteria are presented for L-C and R-C phase shifting networks that develop a constant phase difference over a wide frequency band. The technique is to introduce two phase shifts such that, although they vary with frequency, their difference does not

It is frequently desirable to derive from a given voltage two new voltages of the same frequency but with the phase angle between the new voltages held substantially constant over a wide frequency range, each derived voltage having an amplitude characteristic linearly variable with the amplitude of the input voltage independently of frequency.

To produce the phase difference between the two voltages, two networks whose phase angles increase substantially linearly with the logarithm of the frequency are used. Thus, if the two networks are properly matched, the phase difference between them remains nearly constant over a wide range of frequency.

Network Development

One way of producing the two voltages is to derive both of them from a single source and arrange that the output of either channel is independent of frequency, but that both have phase angles with respect to the input voltages varying in such a way that over a wide band of frequencies

$$\phi_1 - \phi_2 = K \qquad (1)$$

where ϕ_1 = phase angle of output No. 1
ϕ_2 = phase angle of output No. 2
K = a constant
ϕ_1 and ϕ_2 are each their own function of f, the frequency

One possible configuration for these functions is

$$\phi_1 = C + \log f \qquad 2$$

$$\phi_2 = C + \log kf \qquad 3$$

where C and k are constants

Substituting Eq. 2 and 3 in Eq. 1

$$\begin{aligned}\phi_1 - \phi_2 &= C + \log f - C - \log kf \\ &= \log f - \log f - \log k \\ &= -\log k = K \qquad (4)\end{aligned}$$

Thus it is only necessary to find a network configuration which will yield a phase angle which varies as the logarithm of the frequency over a wide range of frequency. The network must also have no change in output amplitude with frequency. The latter limitation usually restricts the final network configuration to lattice types because finite ladder types with any phase shift must be accompanied by amplitude variations. In order to avoid the use of transformers in the lattice structure, a half-lattice will be chosen with the input terminals excited by two equal voltages 180 degrees out of phase. Such voltages are readily available from vacuum tube phase inverters consisting of a tube with equal cathode and anode loads.

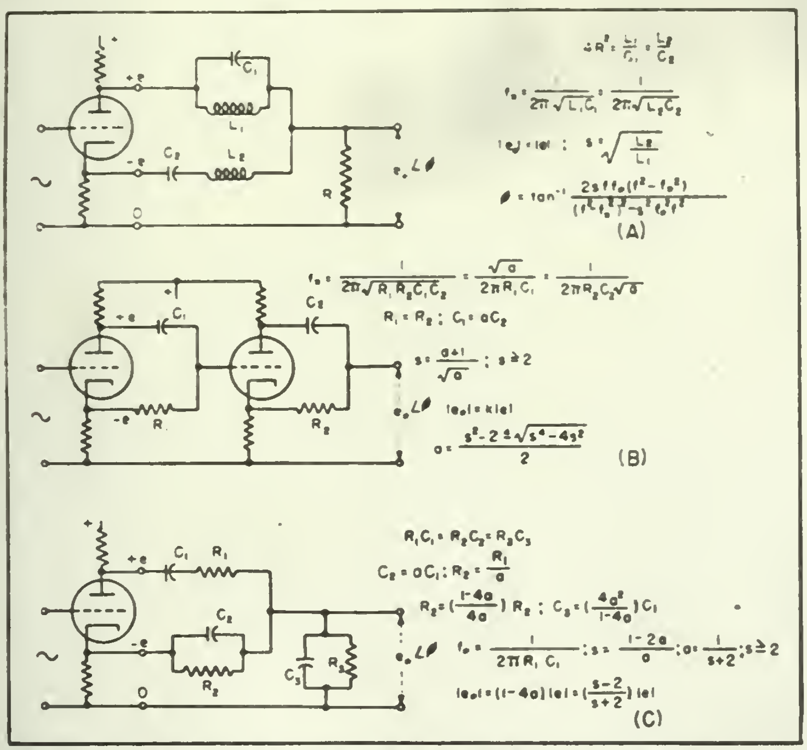


FIG. 1—Three networks for producing phase shift. Networks are used in pairs to obtain two outputs which have a constant phase difference over a wide range of frequency. Phase shift equation given for network of Fig. 1A holds for other two networks

L-C Lattice

A circuit having enough independent parameters to permit the designer to shape the phase angle curve to the required logarithmic form and the basic design equations of the circuit are shown in Fig. 1A. It will be noted from the equation for the phase angle (Fig. 1A) that an arbitrary factor s is included. The choice of s is up to the designer.

If s lies between 3 and 5, a fairly straight line for ϕ is obtained when plotted against a logarithmic frequency scale. As will be shown later, if a second curve for a second similar network, but with a resonant frequency f_o which is 4.53 times the resonant frequency of the first curve, is plotted, the phase

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Phase Shift Networks

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> **Applications**
>
> THE wideband phase shift networks described herein are applicable to
>
> Single sideband telephony accomplished directly at the final carrier frequency without multiple modulators or sharp cutoff filters.
>
> High efficiency broadcast transmitters radiating carrier, upper, and lower sidebands from three separate antennas.
>
> Frequency variation of crystal controlled carrier of communication transmitters either for carrier control or frequency-shift keying.
>
> Circular display on cathode-ray tubes over a wide frequency band without adjustment of the phase shift network.
>
> Variable speed operation of three-phase a-c motors.

angle difference between these two curves is maintained within 90 ±4 degrees over a fairly wide range of frequency if $s = 4$, and is quite satisfactory for voice frequencies because the upper to lower frequency ratio is approximately 28 to 1, as for example would occur in a band from 130 to 3,600 cps.

The band can be widened indefinitely by adding more elements to the bridge arms with the result that the frequency range is expanded more until remarkably good performance is obtained over the full audio range from 30 to 15,000 cps.

The use of inductances in filters is to be avoided if possible because of the inability to obtain a pure inductance; there are always the inevitable series resistance and shunt capacitance. Other objections to the use of inductors are that they may pick up unwanted signals from stray magnetic fields and that they are not constant in inductance with applied voltage. As a consequence a resistance-capacitance type of half-lattice network would be preferable providing the required type of phase curve were obtainable from them.

R-C Lattice

In order to provide a wide range of operation, two simple resistance-capacitance networks can be used in cascade with isolating tubes between them as shown in Fig. 1B. A single network has restricted bandwidth. No terminating resistor can be used in this type of network.

The phase angle of the final output voltage can be adjusted exactly as in the L-C circuit of Fig. 1A because the phase angle equation is identical. The only limitation with this network is that s must be greater than 2, however as previously found, for best results s should be approximately 4, and hence lies well above 2.

The number of stages cascaded in cascade may, of course, be increased to any desired number. When six such stages are used, the frequency range for which a phase angle difference between two sets of networks can be maintained within 90 [illegible] degrees becomes approximately 200 to 1, or [illegible] at audio frequencies from [illegible] 50 to 10,000 cps which is [illegible] for high quality broadcast service.

Another R-C network which does not require [illegible] isolation tubes is shown in Fig. 1C. This network somewhat resembles the L-C network of Fig. 1A but differs in output voltage. The output voltage is a proper fraction of the input voltage. The factor [illegible] must be less than 0.25 or s must be greater than 2 when using this network. As previously pointed out, for best results s should be approximately 4, or $a = 0.167$ which will yield an output voltage of about 0.33 times the input voltage.

The phase angle equation (Fig. 1A) is seen to be identical to those for the previously described networks. In order to find the proper values for f in the two networks which are to yield a phase angle difference of 90 degrees, assume a geometric mean frequency between the upper and lower frequency limits of usable phase angle difference. This mean frequency is not critical, but a value of 700 cps has been suggested as a practical frequency (O. B. Hansen, Down to Earth on 'High Fidelity', Radio Technical Planning Board, Report on Standards and Frequency Allocations for Post-War FM Broadcasting, Panel 5, Jan. 1, 1944, Section VIII). Having decided on the mean frequency of the system, it is merely necessary to design the two phase shift networks so that one has a phase angle of 180 — 45 degrees at 700 cps, and the other a phase angle of 180 [illegible] 45 degrees at 700 cps.

Basis for Design

Suppose the value for f of the first network is to be determined then, designating f of the first network as f_0, and letting f equal the mean frequency, designated as F, the phase angle equation (Fig. 1) becomes

$$\tan \phi = \frac{2 F f_0 [illegible]}{(F^2 - f_0)^2 [illegible]}$$

Setting ϕ = 135 degrees and s = 4 clearing fractions and solving for f gives f = 2.126F, which, for F = 700 cps, gives f = 1,488 cps.

By reciprocal relationships, the value of f, the f_0 of the second network, is f = F/2.126; at F = 700 cps, f = 329 cps.

The parameters of the two networks can now be calculated. For convenience and to establish values of impedance into which vacuum tubes can work satisfactorily, assume that R_1 = 20,000 ohms in each

Table I—Determination of Network Parameters

First Network	Second Network
$f_{01} = 1188$ cps	$f_{02} = 329$ cps
$s = 1.00$	$s = 1.00$
$a = 0.1666$	$a = 0.1666$
$R_1 = 20{,}000$ ohms	$R_1 = 20{,}000$ ohms
$C_1 = \dfrac{1}{2\pi f_0 R_1} = \dfrac{1}{2\pi 1188 \times 20000} = 0.00535\ \mu f$	$C_1 = \dfrac{1}{2\pi f_0 R_1} = \dfrac{1}{2\pi 329 \times 20000} = 0.0242\ \mu f$
$C_2 = a C_1 = 0.166 \times 0.00535 = 0.000892\ \mu f$	$C_2 = a C_1 = 0.166 \times 0.0242 = 0.00403\ \mu f$
$C_3 = \left(\dfrac{4a^2}{1-4a}\right) C_1 = 0.333\ C_1 = 0.333 \times 0.00535 = 0.001783\ \mu f$	$C_3 = \left(\dfrac{4a^2}{1-4a}\right) C_1 = 0.333\ C_1 = 0.333 \times 0.0242 = 0.00806\ \mu f$
$R_2 = \dfrac{R_1}{a} = \dfrac{20{,}000}{0.166} = 120{,}000$ ohms	$R_2 = \dfrac{R_1}{a} = \dfrac{20{,}000}{0.166} = 120{,}000$ ohms
$R_3 = \left(\dfrac{1-4a}{4a}\right) R_2 = \left(\dfrac{1-0.666}{0.666}\right) 120{,}000 = 60{,}000$ ohms	$R_3 = \left(\dfrac{1-4a}{4a}\right) R_2 = \left(\dfrac{1-0.666}{0.666}\right) 120{,}000 = 60{,}000$ ohms

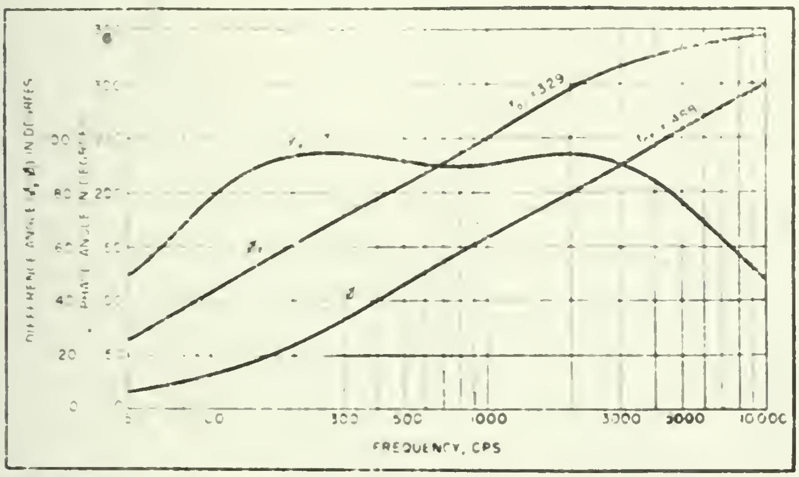

FIG. 2—Phase shift characteristics of networks whose values are given in Table I

network. The calculations are summarized in Table I.

The phase shift curves for these two networks are shown in Fig. 2. Note the long frequency range over which these curves are substantially parallel. The difference angle is plotted to facilitate comparison. Note that this difference angle holds fairly close to 90 degrees over a range of frequencies from 130 to 3,600 cps. This range is quite adequate for a voice frequency channel.

Such a network can be used to build a simple single sideband transmitter without the necessity of using sharp cut-off filters and double or triple modulation. This simplification may be done by combining the outputs of two balanced modulators: one balanced modulator is fed with radio frequency cos ωt and audio frequency $m$cos $(\mu t - \theta)$, and the other balanced modulator is fed with radio frequency cos $(\omega t \pm \pi/2)$ and audio frequency $m$cos $(\mu t - \theta \pm \pi/2)$. If the plus signs are used and the balanced modulator outputs are added, the resultant output would be $m$cos $[(\omega - \mu)t + \theta]$, which is the lower sideband.

Single Sideband Telephony

An experimental system of this type has been constructed and operated using carrier frequencies in the broadcast band. No untoward difficulty was encountered and results were fully up to expectations. The transmitter was modulated with audio signals taken from the output of a broadcast receiver. The single sideband transmission was received by a second broadcast receiver into whose input terminals was also fed an unmodulated continuous wave radio frequency from a signal generator to furnish the missing carrier. The signal generator frequency was adjusted until the reproduced program sounded most natural. No auxiliary filters were used in the radio section of the transmitter to aid in reducing the unwanted sideband.

The question may arise as to how much of the unwanted sideband is permitted to get through the system if not exactly 90 degrees phase difference is obtained between the two audio channels. The ratio of the weaker sideband to the stronger sideband is given by the equation

$$\text{RATIO} = \sqrt{\frac{1 - \cos\delta}{1 + \cos\delta}} \cong \frac{\delta}{2}$$

where δ is the deviation of the difference phase angle from $\pi/2$, with δ expressed in radians for the approximate formula. A deviation from 90 degrees of 6 degrees is required before the weaker sideband becomes 5 percent as strong as the stronger sideband, so that the phase angle curve of Fig. 2 is probably commercially useful over a range in frequencies included between 84 degrees on the low side of the center to 84 degrees on the high side of the center.

A second application for the system is that of providing a closer-to-zero frequency single sideband telephone or broadcast transmission when filters are used. For example, a single sideband transmitter may be constructed along conventional lines with a quartz crystal type of

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band-pass filter to operate on 100 kc. Suppose the pass band extends from 100,600 to 108,000 cps. It is seen that frequencies below 600 cps are attenuated in such a system. Now if the single sideband system described in the present paper were to be used in conjunction with a filter, the filter could transmit from 100,000 to 108,000 cps, and the region between 27 and 600 cps could be taken care of by the audio phase shift system, letting the filter remove the undesired sideband from 99,400 down to 92,000 cps. Thus the resultant radiation could contain single sideband components corresponding to an audio frequency range of 27 to 8,000 cps instead of being limited to a range of 600 to 8,000 cps. The addition of the 27 to 600-cps range will add considerably to the naturalness of male speech and organ music. Frequencies between 0 and 27 cps should be removed from the original audio being fed into the system.

High Efficiency Transmitter

A high efficiency broadcast transmitter can be constructed by employing three power amplifiers and three antennas arranged so that an unmodulated carrier is radiated on a central antenna and the upper and lower sidebands are radiated respectively on two side antennas, which are on a straight line through the central antenna but on opposite sides equidistant from it. The sidebands are generated in much the same manner as described in the preceding section. Only one set of audio phase-shift filters are required, for the upper and lower sidebands can be obtained by simply adjusting the phases of the radio frequencies fed into the two sets of balance modulators.

This system of transmission could be accomplished with the following installed relative power capacity

Carrier	1.00
Upper Sideband	0.25
Lower Sideband	0.25
Total	1.50

When modulating 100 percent, this system may have an overall efficiency of about 66 percent, resulting in a d-c power consumption of 2.25 maximum. This consumption [illegible] down to 1.5 for zero modulation. For an average modulation of 50 percent the power consumption would thus be about 1.875.

The conventional double sideband plus carrier class B amplifier [illegible] an installed power capacity of 4.0 for handling the peaks of 100 percent positive modulation and [illegible] at 33 percent efficiency, requiring a continuous input power of 3.0 from the d-c supply. Thus the saving in d-c power consumption would be about 1.125 based on the carrier rating, or for a 50-kw transmitter the saving would amount to 56.25 kw. Besides the saving in power, the system would have lower [illegible] in the water cooling system, power rectifiers and transformers, and in high power vacuum tubes. The replacement tube cost also would be less.

In the operation of emergency communications transmitters it is often desirable to change the radiated carrier frequency a few hundred or a few thousand cycles to avoid either intentional or accidental jamming. Yet it is also desirable to retain the benefits of precise frequency control such as is provided by a quartz crystal. One solution is to have available a large number of crystals having closely adjacent frequencies selectable by a rotary switch. Another scheme might be to adjust the air gap or shunt capacitance across the crystal. All of these methods have their objectionable features.

Adjustable Carrier Frequency

An alternative arrangement is to make use of the single sideband generator already described. By way of example, suppose the assigned frequency of a transmitter were 4,500 kc. One could then use a crystal oscillator on 4,498 kc in conjunction with the balanced modulator and a 2-kc variable frequency oscillator to produce the required 4,500-kc carrier. The oscillator could then be adjusted to [illegible]

Frequency Shift Keying

[illegible]

Other Applications

[illegible]

By [illegible] can be developed from a single phase source. Such a circuit could be used to control the speed of a three phase synchronous motor. Varying the frequency of the original source would vary the motor speed.

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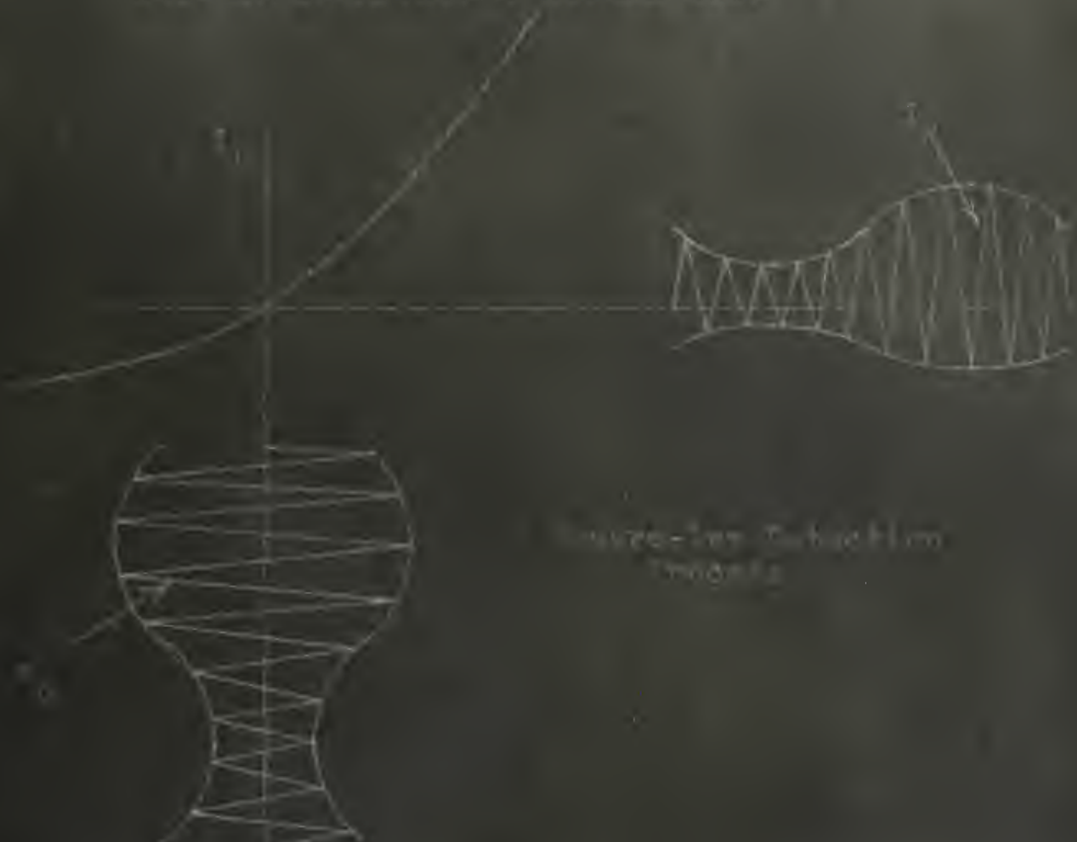
APPENDIX III

DERIVATION OF FREQUENCY COMPONENTS RESULTING FROM SQUARE LAW-DETECTION

Case I. Detection of normal A.M.



$$e_0 = E_c \cos \omega_c t + \tfrac{1}{2} E_c M \cos(\omega_c + \omega_s)t + \tfrac{1}{2} E_c M \cos(\omega_c - \omega_s)t$$

$$i_p = a_1 e_0 + a_2 e_0^2$$

$$i_p = a_1 \left\{ e_0 \right\} + a_2 E_c^2 \left\{ \cos^2 \omega_c t \right.$$
$$+ \frac{M^2}{4} \cos^2 (\omega_c + \omega_s) t + \frac{M^2}{4} \cos^2 (\omega_c - \omega_s) t$$
$$+ M \cos \omega_c t \cos (\omega_c + \omega_s) t$$
$$+ M \cos \omega_c t \cos (\omega_c - \omega_s) t$$
$$\left. + \frac{M^2}{2} \cos (\omega_c + \omega_s) t \cos (\omega_c - \omega_s) t \right\}$$

$$i_p = a_1 \left\{ e_0 \right\} + a_2 E_c^2 \left\{ \frac{1}{2} \left[1 + \cos 2 \omega_c t \right] \right.$$
$$+ \frac{M^2}{8} \left[1 + \cos 2 (\omega_c + \omega_s) t \right]$$
$$+ \frac{M^2}{8} \left[1 + \cos 2 (\omega_c - \omega_s) t \right]$$
$$+ \frac{M}{2} \left[\cos (\omega_c - \omega_c - \omega_s) t + \cos (\omega_c + \omega_c + \omega_s) t \right]$$
$$+ \frac{M}{2} \left[\cos (\omega_c - \omega_c + \omega_s) t + \cos (\omega_c + \omega_c - \omega_c) t \right]$$
$$+ \frac{M^2}{4} \left[\cos (\omega_c + \omega_s - \omega_c + \omega_s) t + \cos (\omega_c + \omega_s + \omega_c - \omega_s) t \right]$$

$$i_p = a_1 \left\{ e_0 \right\} + a_2 E_c^2 \left(\frac{1}{2} + \frac{M^2}{8} + \frac{M^2}{8} \right)$$
$$+ a_2 E_c^2 \left\{ \frac{1}{2} \cos 2 \omega_c t + \frac{M^2}{8} \cos 2 (\omega_c + \omega_s) t \right.$$
$$+ \frac{M^2}{8} \cos 2 (\omega_c - \omega_s) t + \frac{M}{2} \cos \omega_s t$$
$$+ \frac{M}{2} \cos (2 \omega_c + \omega_s) t + \frac{M}{2} \cos \omega_s t$$
$$+ \frac{M}{2} \cos (2 \omega_c - \omega_s) t + \frac{M^2}{4} \cos 2 \omega_s t$$
$$\left. + \frac{M^2}{4} \cos 2 \omega_c t \right\}$$

$$
\begin{aligned}
i_p = {} & a_2 E_c^2\left(\frac{1}{2}+\frac{M^2}{4}\right) + a_2 E_c^2 M \cos \omega_s t \\
& + a_2 E_c^2 \frac{M^2}{4} \cos 2\omega_s t + a_1 E_c \cos \omega_c t \\
& + a_2 E_c^2 \left(\frac{1}{2}+\frac{M^2}{4}\right) \cos 2\omega_c t \\
& + \frac{1}{2} a_1 E_c M \cos(\omega_c+\omega_s)t + \frac{1}{2} a_1 E_c M \cos(\omega_c-\omega_s)t \\
& + a_2 E_c^2 \frac{M^2}{8} \cos 2(\omega_c+\omega_s)t + a_2 E_c^2 \frac{M^2}{8} \cos 2(\omega_c-\omega_s)t \\
& + a_2 E_c^2 \frac{M}{2} \cos(2\omega_c+\omega_s)t + a_2 E_c^2 \frac{M}{2} \cos(2\omega_c-\omega_s)t
\end{aligned}
$$

Components of i_p:

(1.) $a_2 E_c^2\left(\frac{1}{2}+\frac{M^2}{4}\right)$ -- D. C. Component

(2) $a_2 E_c^2 M \cos \omega_s t$ -- Signal Frequency Component

(3) $a_2 E_c^2 \frac{M^2}{4} \cos 2\omega_s t$ -- Double Signal Frequency Component

(4) $a_1 E_c \cos \omega_c t$ -- Carrier Frequency Component

(5) $a_2 E_c^2\left(\frac{1}{2}+\frac{M^2}{4}\right) \cos 2\omega_c t$ -- Double Carrier Frequency Component

(6) $\frac{1}{2} a_1 E_c M \cos(\omega_c+\omega_s)t$ -- Upper Sideband Component

(7) $\frac{1}{2} a_1 E_c M \cos(\omega_c-\omega_s)t$ -- Lower Sideband Component

(8) $a_2 E_c^2 \frac{M^2}{8} \cos 2(\omega_c+\omega_s)t$ -- Double Upper Sideband Frequency Component

(9) $a_2 E_c^2 \frac{M^2}{8} \cos 2(\omega_c-\omega_s)t$ -- Double Lower Sideband Frequency Component

(10) $a_2 E_c^2 \frac{M}{2} \cos(2\omega_c+\omega_s)t$ -- Double Carrier Plus Signal Frequency Component

(11) $a_2 E_c^2 \frac{M}{2} \cos(2\omega_c-\omega_s)t$ -- Double Carrier Minus Signal Frequency Component

Case II. Detection of A.M. with carrier suppressed at transmitter and reinserted at receiver at a frequency of $\omega_c + \Delta$.

$$e_o = C\cos(\omega_c+\Delta)t + \tfrac{1}{2}E_cM\cos(\omega_c+\omega_s)t + \tfrac{1}{2}E_cM\cos(\omega_c-\omega_s)t$$

$$i_p = a_1 e_o + a_2 e_o^2$$

$$\begin{aligned} i_p = a_1\{e_o\} + a_2\Big\{&C^2\cos^2(\omega_c+\Delta)t \\ &+ \frac{E_c^2M^2}{4}\cos^2(\omega_c+\omega_s)t + \frac{E_c^2M^2}{4}\cos^2(\omega_c-\omega_s)t \\ &+ CE_cM\cos(\omega_c+\Delta)t\cos(\omega_c+\omega_s)t \\ &+ CE_cM\cos(\omega_c+\Delta)t\cos(\omega_c-\omega_s)t \\ &+ \frac{E_c^2M^2}{2}\cos(\omega_c+\omega_s)t\cos(\omega_c-\omega_s)t\Big\} \end{aligned}$$

$$\begin{aligned} i_p = {} & a_2\left(\frac{C^2}{2}+\frac{E_c^2M^2}{4}\right) + \frac{a_2CE_cM}{2}\cos(\omega_s-\Delta)t \\ &+ \frac{a_2CE_cM}{2}\cos(\omega_s+\Delta)t + \frac{a_2E_c^2M^2}{4}\cos 2\omega_s t \\ &+ a_1C\cos(\omega_c+\Delta)t + a_2\frac{E_c^2M^2}{4}\cos 2\omega_c t \\ &+ a_2\frac{C^2}{2}\cos 2(\omega_c+\Delta)t + a_1\frac{E_cM}{2}\cos(\omega_c+\omega_s)t \\ &+ a_1\frac{E_cM}{2}\cos(\omega_c-\omega_s)t + a_2\frac{E_c^2M^2}{8}\cos 2(\omega_c+\omega_s)t \\ &+ a_2\frac{E_c^2M^2}{8}\cos 2(\omega_c-\omega_s)t + a_2\frac{CE_cM}{2}\cos(2\omega_c+\Delta+\omega_s)t \\ &+ a_2\frac{CE_cM}{2}\cos(2\omega_c+\Delta-\omega_s)t \end{aligned}$$

Components of i_p:

(1) $a_2\left(\frac{C^2}{2} + \frac{E_c^2 M^2}{4}\right)$	-- D. C. Component
(2) $\frac{a_2}{2} C E_c M \cos(\omega_s - \Delta)t$	-- Signal Frequency Minus Δ
(3) $\frac{a_2}{2} C E_c M \cos(\omega_s + \Delta)t$	-- Signal Frequency Plus Δ
(4) $\frac{a_2}{4} E_c^2 M^2 \cos 2\omega_s t$	-- Double Signal Frequency
(5) $a_1 C \cos(\omega_c + \Delta)t$	-- Reinserted Carrier Frequency
(6) $\frac{a_2}{4} E_c^2 M^2 \cos 2\omega_c t$	-- Double Original Carrier Frequency
(7) $\frac{a_2 C^2}{2} \cos 2(\omega_c + \Delta)t$	-- Double Reinserted Carrier Frequency
(8) $\frac{a_1 E_c M}{2} \cos(\omega_c + \omega_s)t$	-- Upper Sideband
(9) $\frac{a_1 E_c M}{2} \cos(\omega_c + \omega_s)t$	-- Lower Sideband
(10) $\frac{a_2}{8} E_c^2 M^2 \cos 2(\omega_c + \omega_s)t$	-- Double Upper Side-band Frequency
(11) $\frac{a_2}{8} E_c^2 M^2 \cos 2(\omega_c - \omega_s)t$	-- Double Lower Side-band Frequency
(12) $\frac{a_2}{2} C E_c M \cos(2\omega_c + \Delta + \omega_s)t$	-- Reinserted Carrier Plus Upper Side-band frequency
(13) $\frac{a_2}{2} C E_c M \cos(2\omega_c + \Delta - \omega_s)t$	-- Reinserted Carrier Plus Lower Side-band Frequency

Note that components (2) and (3) may be combined to give

$$a_2 C E_c M \cos \Delta t \cos \omega_s t$$

which is a signal similar to the audio signal of straight A.M., detected, but varying in amplitude from zero to a_2CE_cM at the rate Δ. The result would be unintelligible.

Case III. Detection of A.M. with carrier suppressed at transmitter and reinserted at receiver at the original frequency but differing in phase by θ°.

$$e_o = C\cos(\omega_c t + \theta) + \frac{E_c M}{2}\cos(\omega_c + \omega_s)t + \frac{E_c M}{2}\cos(\omega_c - \omega_s)t$$

$$i_p = a_1 e_o + a_2 e_o^2$$

$$\begin{aligned} i_p = a_1\{e_o\} + a_2\Big\{ & C^2\cos^2(\omega_c t + \theta) \\ & + \frac{E_c^2 M^2}{4}\cos^2(\omega_c + \omega_s)t + \frac{E_c^2 M^2}{4}\cos^2(\omega_c - \omega_s)t \\ & + C E_c M\cos(\omega_c t + \theta)\cos(\omega_c + \omega_s)t \\ & + C E_c M\cos(\omega_c t + \theta)\cos(\omega_c - \omega_s)t \\ & + \frac{E_c^2 M^2}{2}\cos(\omega_c + \omega_s)t\cos(\omega_c - \omega_s)t\Big\} \end{aligned}$$

$$\begin{aligned} i_p = {} & a_2\left(\frac{C^2}{2} + \frac{E_c^2 M^2}{4}\right) + \frac{a_2 C E_c M}{2}\cos(\omega_s t - \theta) \\ & + \frac{a_2 C E_c M}{2}\cos(\omega_s t + \theta) + \frac{a_2 E_c^2 M^2}{4}\cos 2\omega_s t \\ & + a_1 C\cos(\omega_c t + \theta) + \frac{a_1 E_c M}{2}\cos(\omega_c + \omega_s)t \\ & + \frac{a_1 E_c M}{2}\cos(\omega_c - \omega_s)t + \frac{a_2 C^2}{2}\cos 2(\omega_c t + \theta) \\ & + \frac{a_2 E_c^2 M^2}{8}\cos 2(\omega_c + \omega_s)t + \frac{a_2 E_c^2 M^2}{8}\cos 2(\omega_c - \omega_s)t \\ & + \frac{a_2 C E_c M}{2}\cos(2\omega_c t + \theta + \omega_s t) \\ & + \frac{a_2 C E_c M}{2}\cos(2\omega_c t + \theta - \omega_s t) + \frac{a_2 E_c^2 M^2}{4}\cos 2\omega_c t \end{aligned}$$

Components of i_p:

(1) $a_2\left(\frac{C^2}{2} + \frac{E_c^2 M^2}{4}\right)$	--	D.C. Component
(2) $\frac{a_2}{2} C E_c M \cos(\omega_s t - \theta)$	--	Signal Frequency With Negative Phase Shift
(3) $\frac{a_2}{2} C E_c M \cos(\omega_s t + \theta)$	--	Signal Frequency With Positive Phase Shift
(4) $\frac{a_2}{4} E_c^2 M^2 \cos 2\omega_s t$	--	Double Signal Frequency
(5) $a_1 C \cos(\omega_c t + \theta)$	--	Reinserted Carrier
(6) $\frac{a_2}{4} E_c^2 M^2 \cos 2\omega_c t$	--	Double Original Carrier Frequency
(7) $\frac{a_2 C^2}{2} \cos 2(\omega_c t + \theta)$	--	Double Reinserted Carrier Frequency
(8) $\frac{a_1}{2} E_c M \cos(\omega_c + \omega_s)t$	--	Upper Sideband
(9) $\frac{a_1}{2} E_c M \cos(\omega_c - \omega_s)t$	--	Lower Sideband
(10) $\frac{a_2}{8} E_c^2 M^2 \cos 2(\omega_c + \omega_s)t$	--	Double Upper Side-band Frequency
(11) $\frac{a_2}{8} E_c^2 M^2 \cos 2(\omega_c - \omega_s)t$	--	Double Lower Side-band Frequency
(12) $\frac{a_2}{2} C E_c M \cos(2\omega_c t + \theta + \omega_s t)$	--	Reinserted Carrier Plus Upper Sideband Frequency
(13) $\frac{a_2}{2} C E_c M \cos(2\omega_c t + \theta - \omega_s t)$	--	Reinserted Carrier Plus Lower Sideband Frequency

Note that components (2) and (3) may be combined to give

$$a_2 C E_c M \cos\theta \cos\omega_s t,$$

which is equal to signal component of Case I, but multiplied by cos θ, which for $\theta \neq 0$, will reduce the value of this component.

Case IV. Detection of A.M. with carrier suppressed at transmitter and not reinserted.

$$e_0 = \frac{E_c M}{2} \cos(\omega_c + \omega_s)t + \frac{E_c M}{2} \cos(\omega_c - \omega_s)t$$

$$i_p = a_1 e_0 + a_2 e_0^2$$

$$i_p = a_1\{e_0\} + a_2 \frac{E_c^2 M^2}{4} \{\cos^2(\omega_c + \omega_s)t + \cos^2(\omega_c - \omega_s)t + 2\cos(\omega_c + \omega_s)t \cos(\omega_c - \omega_s)t\}$$

$$i_p = \frac{a_2 E_c^2 M^2}{4} + \frac{a_2 E_c^2 M^2}{4} \cos 2\omega_s t + \frac{a_1 E_c M}{2} \cos(\omega_c + \omega_s)t + \frac{a_1 E_c M}{2} \cos(\omega_c - \omega_s)t + \frac{a_2 E_c^2 M^2}{8} \cos 2(\omega_c + \omega_s)t + \frac{a_2 E_c^2 M^2}{8} \cos 2(\omega_c - \omega_s)t + \frac{a_2 E_c^2 M^2}{4} \cos 2\omega_c t$$

Components of i_p:

(1) $\frac{a_2 E_c^2 M^2}{4}$ -- D. C. Component

(2) $\frac{a_2 E_c^2 M^2}{4} \cos 2\omega_s t$ -- Double Signal Frequency

(3) $\frac{a_2 E_c^2 M^2}{4} \cos 2\omega_c t$ -- Double Carrier Frequency

(4) $\frac{a_1 E_c M}{2} \cos(\omega_c + \omega_s)t$ -- Upper Sideband

(5) $\frac{a_1 E_c M}{2} \cos(\omega_c - \omega_s)t$ -- Lower Sideband

(6) $\frac{a_2 E_c^2 M^2}{8} \cos 2(\omega_c + \omega_s)t$ -- Double Upper Sideband Frequency

(7) $\frac{a_2 E_c^2 M^2}{8} \cos 2(\omega_c - \omega_s)t$ -- Double Lower Sideband Frequency

Case V. Detection of SSSC with no carrier reinsertion.

$$e_o = \frac{E_c M}{2} \cos(\omega_c + \omega_s)t$$

$$i_p = a_1 e_o + a_2 e_o^2$$

$$i_p = \frac{a_1 E_c M}{2} \cos(\omega_c + \omega_s)t + \frac{a_2 E_c^2 M^2}{4} \cos^2(\omega_c + \omega_s)t$$

$$i_p = \frac{a_2 E_c^2 M^2}{8} + \frac{a_1 E_c M}{2} \cos(\omega_c + \omega_s)t + \frac{a_2 E_c^2 M^2}{8} \cos 2(\omega_c + \omega_s)t$$

Components of i_p:

(1) $\frac{a_2 E_c^2 M^2}{8}$ -- D.C. Component

(2) $\frac{a_1 E_c M}{2} \cos(\omega_c + \omega_s)t$ -- Upper Sideband Frequency

(3) $\frac{a_2 E_c^2 M^2}{8} \cos 2(\omega_c + \omega_s)t$ -- Lower Sideband Frequency

No signal frequency component is present, thus detection is not possible.

Case VI. Detection of SSSC with carrier reinserted at receiver.

$$e_o = C \cos \omega_c t + \frac{E_c M}{2} \cos(\omega_c + \omega_s) t$$

$$i_p = a_1 e_o + a_2 e_o^2$$

$$i_p = a_1 C \cos \omega_c t + \frac{a_1 E_c M}{2} \cos(\omega_c + \omega_s) t$$
$$+ a_2 C^2 \cos^2 \omega_c t + a_2 \frac{E_c^2 M^2}{4} \cos^2(\omega_c + \omega_s) t$$
$$+ a_2 C E_c M \cos \omega_c t \cos(\omega_c + \omega_s) t$$

$$i_p = \left(\frac{a_2 C^2}{2} + \frac{a_2 E_c^2 M^2}{8}\right) + \frac{a_2 C E_c M}{2} \cos \omega_s t$$
$$+ a_1 C \cos \omega_c t + \frac{a_1 E_c M}{2} \cos(\omega_c + \omega_s) t$$
$$+ \frac{a_2 C E_c M}{2} \cos(2\omega_c + \omega_s) t + \frac{a_2 C^2}{2} \cos 2\omega_c t$$
$$+ \frac{a_2 E_c^2 M^2}{8} \cos 2(\omega_c + \omega_s) t$$

Components of i_p:

(1) $a_2\left(\frac{C^2}{2} + \frac{E_c^2 M^2}{8}\right)$ -- D.C. Component

(2) $\frac{a_2 C E_c M}{2} \cos \omega_s t$ -- Signal Frequency

(3) $a_1 C \cos \omega_c t$ -- Carrier Frequency

(4) $\frac{a_2 C^2}{2} \cos 2\omega_c t$ -- Double Carrier Frequency

(5) $\frac{a_1 E_c M}{2} \cos(\omega_c + \omega_s) t$ -- Upper Sideband Frequency

(6) $\frac{a_2 E_c^2 M^2}{8} \cos 2(\omega_c + \omega_s) t$ -- Double Upper Sideband Frequency

(7) $\frac{a_2 C E_c M}{2} \cos(2\omega_c + \omega_s) t$ -- Double Carrier Plus Signal Frequency

Case VII. Detection of SSSC with carrier reinserted at receiver at a frequency of $(\omega_c + \Delta)$.

$$e_o = C\cos(\omega_c+\Delta)t + \frac{E_c M}{2}\cos(\omega_c+\omega_s)t$$

$$i_p = a_1 e_o + a_2 e_o^2$$

$$i_p = a_1\{e_o\} + a_2\Big\{C^2\cos^2(\omega_c+\Delta)t + \frac{E_c^2M^2}{4}\cos^2(\omega_c+\omega_s)t + CE_cM\cos(\omega_c+\Delta)t\cos(\omega_c+\omega_s)t\Big\}$$

$$\begin{aligned} i_p = {} & \left(\frac{a_2C^2}{2} + \frac{a_2E_c^2M}{8}\right) + \frac{a_2CE_cM}{2}\cos(\omega_c-\Delta)t \\ & + a_1C\cos(\omega_c+\Delta)t + \frac{a_1E_cM}{2}\cos(\omega_c+\omega_s)t \\ & + \frac{a_2CE_cM}{2}\cos(2\omega_c+\Delta+\omega_s)t + \frac{a_2C^2}{2}\cos 2(\omega_c+\Delta)t \\ & + \frac{a_2E_c^2M^2}{8}\cos 2(\omega_c+\omega_s)t \end{aligned}$$

Components of i_p:

(1) $a_2\left(\frac{C^2}{2} + \frac{E_c^2M^2}{8}\right)$ -- D.C. Component

(2) $\frac{a_2CE_cM}{2}\cos(\omega_c-\Delta)t$ -- Signal Frequency Minus Δ

(3) $a_1C\cos(\omega_c+\Delta)t$ -- Reinserted Carrier Frequency

(4) $\frac{a_2C^2}{2}\cos 2(\omega_c+\Delta)t$ -- Double Reinserted Carrier Frequency

(5) $\frac{a_1E_cM}{2}\cos(\omega_c+\omega_s)t$ -- Upper Sideband Frequency

(6) $\frac{a_2E_c^2M^2}{8}\cos 2(\omega_c+\omega_s)t$ -- Double Upper Sideband Frequency

(7) $\frac{a_2CE_cM}{2}\cos(2\omega_c+\Delta+\omega_s)t$ -- Upper Sideband Plus Reinserted Carrier Freq.

APPENDIX IV

DERIVATION OF AUDIO-FREQUENCY COMPONENTS RESULTING FROM DETECTION, BY ENVELOPE ANALYSIS METHOD

Case I. Detection of normal A.M.

Note: In this method of solution, only audio frequency components are significant.

$$i_p = a_1 e_o + a_2 e_o^2$$

For the envelope only,

$i = a_1 e + a_2 e^2$, where e is the envelope voltage.

For the upper envelope, $e_+ = + E_c(1 + M \cos \omega_s t)$.

For the lower envelope, $e_- = - E_c(1 + M \cos \omega_s t)$.

Then

$$i_+ = a_1 E_c (1 + M \cos \omega_s t) + a_2 E_c^2 (1 + M \cos \omega_s t)^2$$
$$i_- = -a_1 E_c (1 + M \cos \omega_s t) + a_2 E_c^2 (1 + M \cos \omega_s t)^2$$

$$i = \frac{i_+ + i_-}{2} = a_2 E_c^2 (1 + M \cos \omega_s t)^2$$

$$i = a_2 E_c^2 (1 + 2M \cos \omega_s t + M^2 \cos^2 \omega_s t)$$

$$i = a_2 E_c^2 + 2 a_2 E_c^2 M \left(\cos \omega_s t + \frac{M}{4} + \frac{M}{4} \cos 2\omega_s t\right)$$

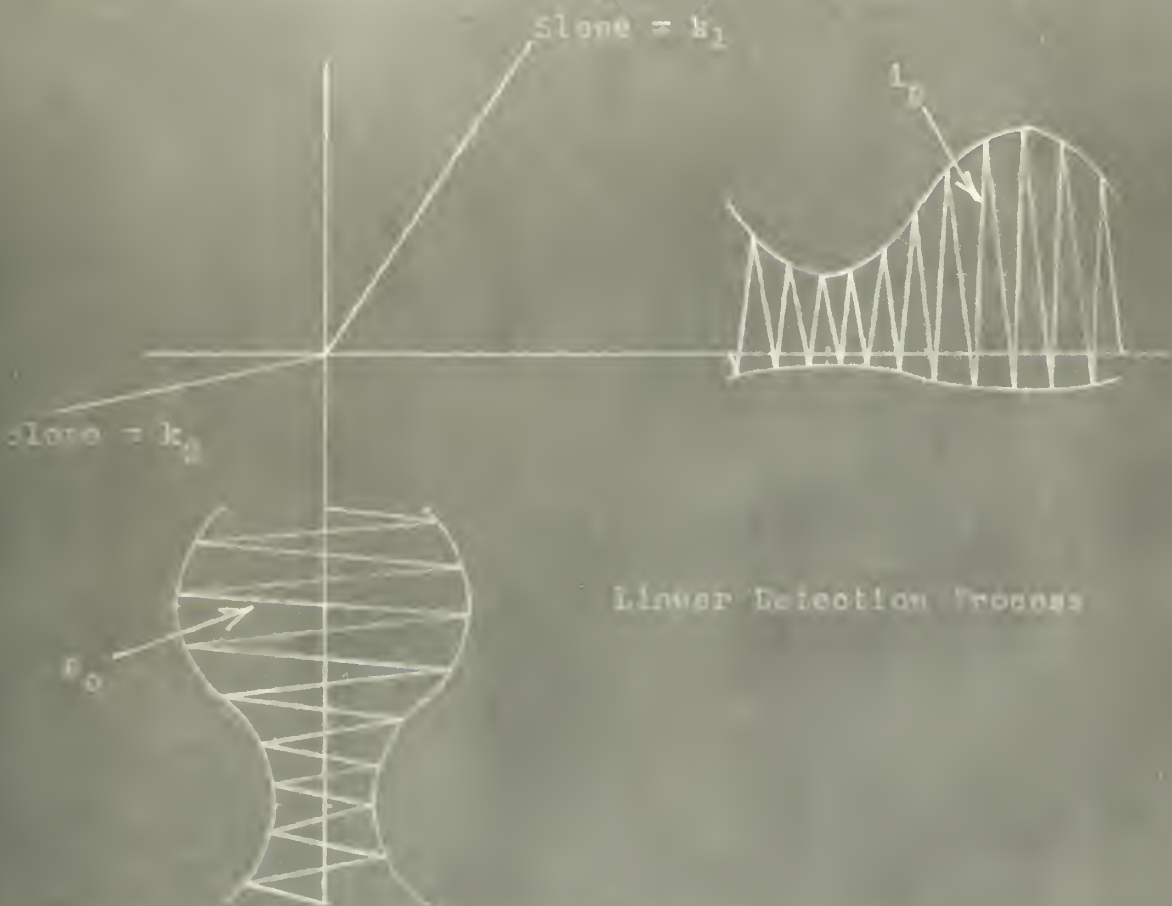
Audio Frequency Components:

(1) $2a_2E_c^2M \cos \omega_s t$ -- Signal Frequency

(2) $1/2\ a_2E_c^2M^2 \cos 2\omega_s t$ -- Double Signal Frequency

Note that the ratio of the above components is M/4, which was the same value obtained in Appendix III, Case I, thus this is seen, by example, to be a satisfactory method for determining the relative magnitudes of audio frequency components resulting from detection.

Case II. Linear Detection of Normal A.M.



For the envelope, $i = ke$, where

$e_+ = + E_c(1 + M \cos \omega_s t)$

$e_- = - E_c(1 + M \cos \omega_s t)$

Then

$i_+ = k_1 E_c(1 + M \cos \omega_s t)$

$i_- = -k_2 E_c(1 + M \cos \omega_s t)$ and

$i = E_c/2\ (k_1 - k_2)(1 + M \cos \omega_s t)$

The only audio frequency component is

$E_c M/2\ (k_1 - k_2) \cos \omega_s t$ -- Signal Frequency.

APPENDIX V. Hetrodyne Detection Analysis

(From "Theory and Applications of Electron Tubes -- H. J. Reich.)

5-14. Ideal Linear Diode Detection of the Sum of Two Voltages of Different Frequencies. The linear diode detector is sometimes used[1] to produce a difference frequency as the result of the simultaneous application of two sinusoidal voltages of

Fig. 5-14. Resultant of two sine waves of equal amplitude and 5 to 4 frequency ratio.

unequal frequency. To analyze this process it is necessary first to study the form of the resultant wave produced by the addition of two sine waves. Figure 5-14 shows the resultant of two sine waves of equal amplitude and 5 to 4 frequency ratio. By adding waves that have both unequal frequency and unequal amplitude, the reader may show that, in general, the resultant of two sine waves of somewhat different frequency resembles an amplitude-modulated wave. It differs from such a wave in that the points at which the resultant crosses the axis are not equally spaced, indicating additional phase modulation.

The form of the envelope of the resultant wave and the manner in which the phase varies with time can be determined analytically. Let the two impressed voltages be $e_1 = E_1 \cos \omega_1 t$ and $e_2 = E_2 \cos \omega_2 t$. The resultant voltage is

$$e = E_1 \cos \omega_1 t + E_2 \cos \omega_2 t \qquad (5\text{-}35)$$

$$= E_1 \cos \omega_1 t + E_2 \cos [\omega_1 t - (\omega_1 - \omega_2) t] \qquad (5\text{-}36)$$

[1] Roder, Hans, *Proc. I. R. E.*, **20**, 1946 (1932).

By [illegible] (5-36) and collecting terms [illegible] the resultant into

$$E \quad [illegible] \tag{5-37}$$

in which

$$E = E_1\sqrt{1 + 2h\cos(\omega_1 - \omega_2)t + h^2} \tag{5-38}$$

$$\phi = \tan^{-1}\frac{h\sin(\omega_1 - \omega_2)t}{1 + h\cos(\omega_1 - \omega_2)t} \tag{5-39}$$

$$h = \frac{E_2}{E_1} \tag{5-40}$$

If e_1 represents the component of the applied voltage that has the larger amplitude, then $E_1 > E_2$, and $h \leq 1$.

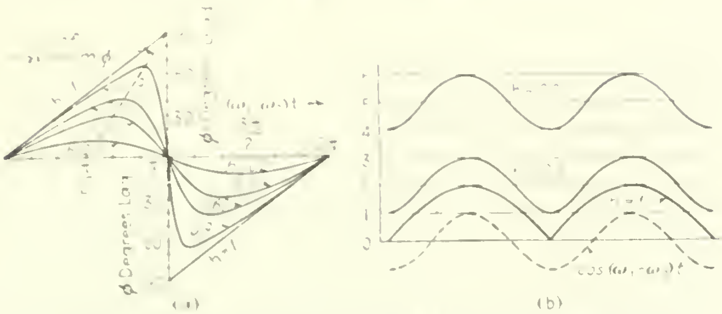


Fig. 5-15. Effect of amplitude ratio h upon (a) phase modulation and (b) form of envelope of the resultant of two sine waves of unequal frequency.

Figure 5-15a shows how the phase angle ϕ varies throughout one cycle of difference frequency, $f_1 - f_2$, for various values of h between 0.2 and 1. When the amplitudes are equal, $h = 1$, and ϕ varies between $+90$ and $-90°$; a sudden reversal of phase takes place at the instant $(\omega_1 - \omega_2)t = \pi$. As h is decreased by changing the amplitude of either component, the maximum value of ϕ rapidly decreases, being only slightly greater than $11\frac{1}{2}°$ when $h = 0.2$. Figure 5-15b shows how E varies throughout the difference-frequency cycle, for various values of h, E_2 being kept constant at unity. Comparison with the dotted curve, which represents $\cos(\omega_1 - \omega_2)t$, shows that the amplitude varies at a fundamental frequency equal to the difference frequency but also contains a steady component and harmonics of the difference frequency. E. B. Moullin has shown[1] that

[1] Moullin, E. B., *Wireless Eng.*, **9**, 378 (1932). See also F. M. Colebrook, *Wireless Eng.*, **9**, 195 (1932).

$$\sqrt{1 + \ldots 2h \cos \ldots (\omega_1 - \omega_2)t} \ldots$$

$$\left(1 + \frac{h^2}{2^2} + \frac{[illegible]}{2^{\ldots}} + \frac{h^{\ldots}}{2^{\ldots}} \ldots\right) + h\left(1 - \frac{[illegible]}{2} \ldots \frac{[illegible]}{2} \ldots \frac{5h^{\ldots}}{2^{\ldots}} \ldots\right)\cos(\omega_1 - \omega_2)t - \frac{h}{4}\left(1 - \frac{h^{\ldots}}{4} - \frac{5h^{\ldots}}{2^{\ldots}} \ldots\right)\cos 2(\omega_1 - \omega_2)t + \frac{h}{8}\left(1 - \frac{5h}{2} \ldots \frac{7h^{\ldots}}{2^{\ldots}} \ldots\right)\cos 3(\omega_1 - \omega_2)t - \frac{5h^{\ldots}}{2^{\ldots}}\left(1 - \frac{7h}{20} \ldots \frac{21h^{\ldots}}{5 \times 2^{\ldots}} \ldots\right)\cos 4(\omega_1 - \omega_2)t + \cdots \quad (5\text{-}41)$$

Each coefficient of this series is in itself an infinite series which converges rapidly if $h < 1$. Considering only the terms given in Eq. (5-41) the series is accurate to within 1 per cent even when $h = 1$. Equation (5-41) proves rigorously that E contains a steady component, a fundamental difference-frequency component, and all harmonics of the difference frequency.

This analysis shows that the resultant of two sine waves of different frequencies is equivalent to a wave whose frequency is that of the component of higher amplitude and whose amplitude and phase are modulated simultaneously. The signal voltage with which the amplitude of this equivalent wave is modulated consists of the fundamental difference frequency and all its harmonics. The phase modulation, the fundamental frequency of which is also equal to the difference frequency, may be made as small as desired by making h small. Under the assumption that h is small enough so that the phase modulation is negligible, the sum of the two sine waves is equivalent to an amplitude-modulated wave $E_1 f(t) \cos \omega_1 t$, in which $f(t)$ is given by Eq. (5-41). In accordance with the principle explained in Sec. 5-13, the application of this voltage to a linear diode detector will result in an output voltage of the form $DE_1 f(t)$, where D is the detection efficiency given by the curves of Fig. 5-12.

An examination of the coefficients of Eq. 5-41 shows that the ratio of the amplitude of the nth harmonic to the amplitude of the fundamental difference-frequency output of the detector is roughly proportional to h^{n-1}.[1] Therefore the harmonic content may be made as small as desired by making the amplitude of one

[1] The exactness of this proportionality increases rapidly as h becomes smaller.

[illegible] of the input much larger than the other. The amplitude [illegible] the fundamental component of detector output voltage is

$$\text{Fund } E_{\ldots} = E_1 Dh\left(1 - \frac{h}{2} + \frac{h}{2} - \frac{5h}{2} + \cdots\right) \qquad (5\text{-}42)$$

As h is decreased, this approaches the limiting value

$$\text{Fund } E_{\ldots} = E_1 Dh = E_2 D \qquad (5\text{-}43)$$

This shows that, if one input voltage is considerably larger than the other, the amplitude of the fundamental difference-frequency component of linear diode detector output depends only upon the amplitude of the smaller input voltage.[1] This is of importance in the design of superheterodyne radio receivers and heterodyne oscillators (see page 351).

5-15. Effect of Curvature of Diode Characteristic. Equations (5-23) and (5-30) hold only for linear characteristics. If the diode characteristic is not linear, the wave of plate current is not an exact replica of the upper half of the applied modulated voltage. This fact suggests that the output contains harmonics of the signal voltage. In general, the relation between total instantaneous diode plate current and plate voltage may be expressed in terms of a series

APPENDIX VI. Construction of a SSSC Adaptor

The adaptor, whose construction is to be described, was designed by the General Electric Company.[2] It was designed to be used with a good quality communications receiver, making possible the reception of SSSC transmissions, the reception of only one sideband (either) of an A.M. transmission, the reception of narrow-band FM transmissions, and the reception of PM transmissions.

As may be seen from the circuit diagram, this adaptor follows the phasing method of sideband separation, together with reinserted (or exalted) carrier and reactance-tube lock-in circuit. A type 6AK6 tube is employed to provide proper impedance match between the adaptor and the receiver I.F. output. An audio stage and a voltage-regulated, highly filtered power supply complete the circuit. Positions 1 and 2 of Switch ABC permit reception on either upper or lower sideband. Position 3 permits reception of both sidebands with the local oscillator giving exalted carrier reception. Position 4 is for normal reception.

The physical lay-out of the components closely followed the circuit diagram arrangement. The 6AK6, with it's associated components was mounted in a small shield can, and located in the receiver chasis. The small filament

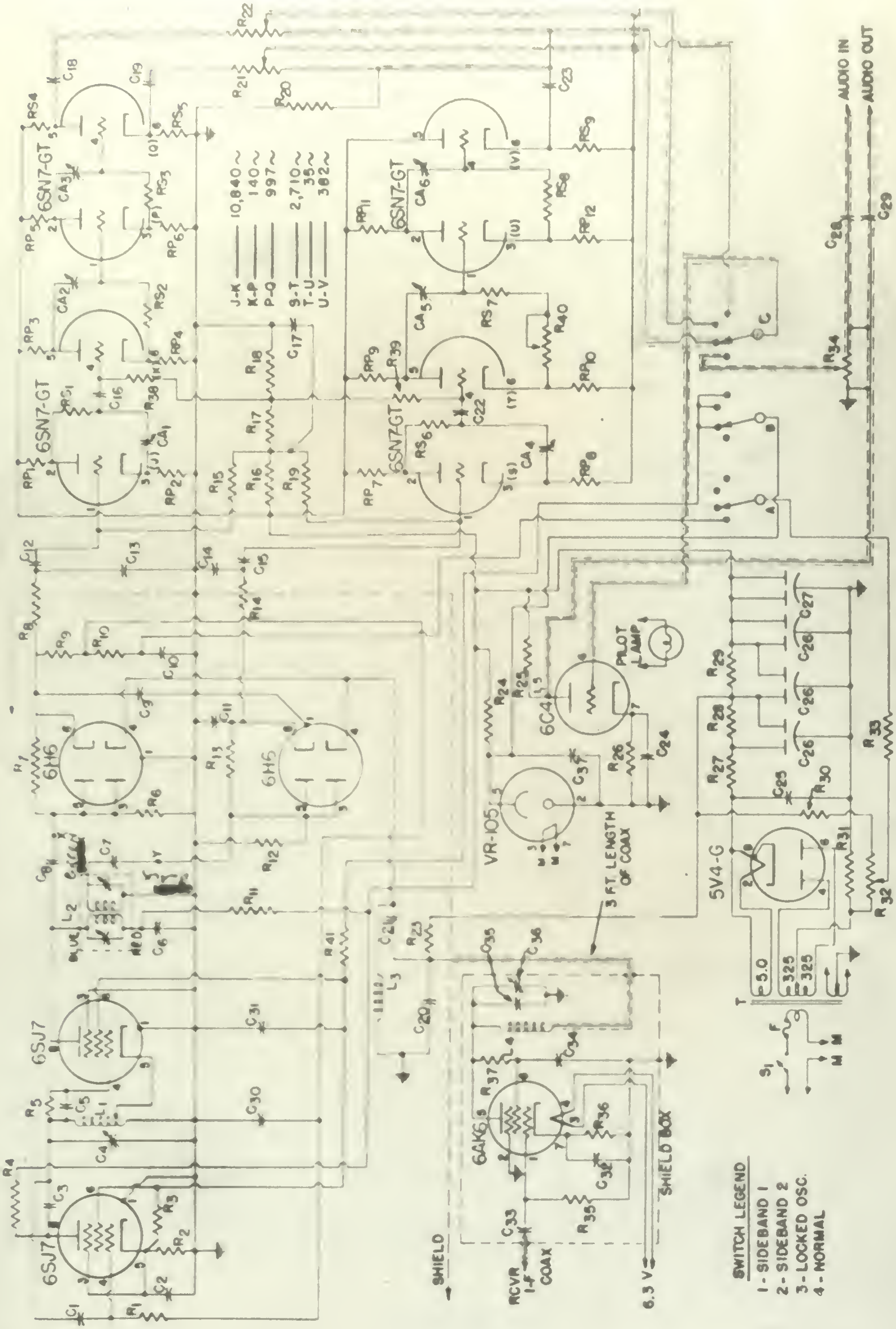


Fig. 2. Circuit Diagram of SSB Adapter

2

current for this tube was easily supplied by the receiver power supply.

Of the components, it should be mentioned that all resistors in the audio phase-shift networks were of the 1%, low-drift type so that accurate phase-shift could be maintained without continual alignment.

The alignment procedure was somewhat complicated. After connecting the adaptor to the receiver (a Hallicrafter S-40A was used), it was first necessary to adjust the trimmer of the final I.F. transformer to compensate for the additional capacitance introduced by the 6AK6 stage. Next, adjustment of C_{36} gave maximum input into the adaptor. In the adaptor constructed, adjustment of L_4 was also necessary. The coupling of L_2, which was a conventional I.F. transformer, had to be increased to that the local oscillator voltages into the two demodulator tubes were equal. Adjustment of the condensers across primary and secondary windings of L_2 was required to obtain accurately, the 90° phase-shift between the two demodulators. An oscilloscope, with horizontal and vertical inputs connected to the two demodulators, served as indicator for this phase aligning. When a perfect circle was obtained, the phasing was correct.

The final stage of alignment was the alignment of the two audio phase-shift networks. Again, the use of an oscilloscope served as the indicator. Each triode section of the 6SN7's with it's component parts was adjusted separately. The frequencies shown on the circuit diagram were those for which the respective elements would give 90° phase-shift. The adjustable elements were the indicated trimmer condensers and the adjustable resistance, R_{40}. After each element was correctly adjusted, the two systems were checked as a whole, over a frequency range of 280 cps to 6000 cps. As was mentioned before, the use of very stable resistors in the phase-shift networks is essential if this alignment procedure is not to be repeated at frequent intervals.

Results were, in general, very good. Attenuation of the undesirable sideband was 35 db below the desired one in reception of an AM transmission. SSSC reception was readily accomplished, but not with the ease that had been anticipated. Probably, the development of a certain degree of skill in the use of this adaptor is necessary to obtain optimum results. The only disappointing feature was the operation of the reactance tube in locking-in the local oscillator to the carrier frequency. A comparatively large carrier input was necessary to obtain satisfactory results.

The local carrier would not quickly synchronize even on A.M. transmissions, if the incoming signal strength was rather weak. This was far short of the desired results of locking on a carrier 20 db below the sideband strength. It is probable that a better physical layout of the reactance tube stage to reduce stray capacitance would improve performance.

The effectiveness of SS reception of AM in reducing interference was quickly appreciated, particularly on communications channels. In several cases, signals were made understandable by use of only one sideband, where they had been entirely unintelligible, because of interference, when both sidebands were being detected.

While certain elements need additional refining, it is believed that the use of this type of an adaptor, even for reception of AM, is highly desirable for communications work. The advantages resulting from use on communications channels are not readily apparent on broadcast reception, because the types of interference are not the same for the two channels.

Thesis 13112
N37 Neman
Single sideband, suppressed carrier radio communication.